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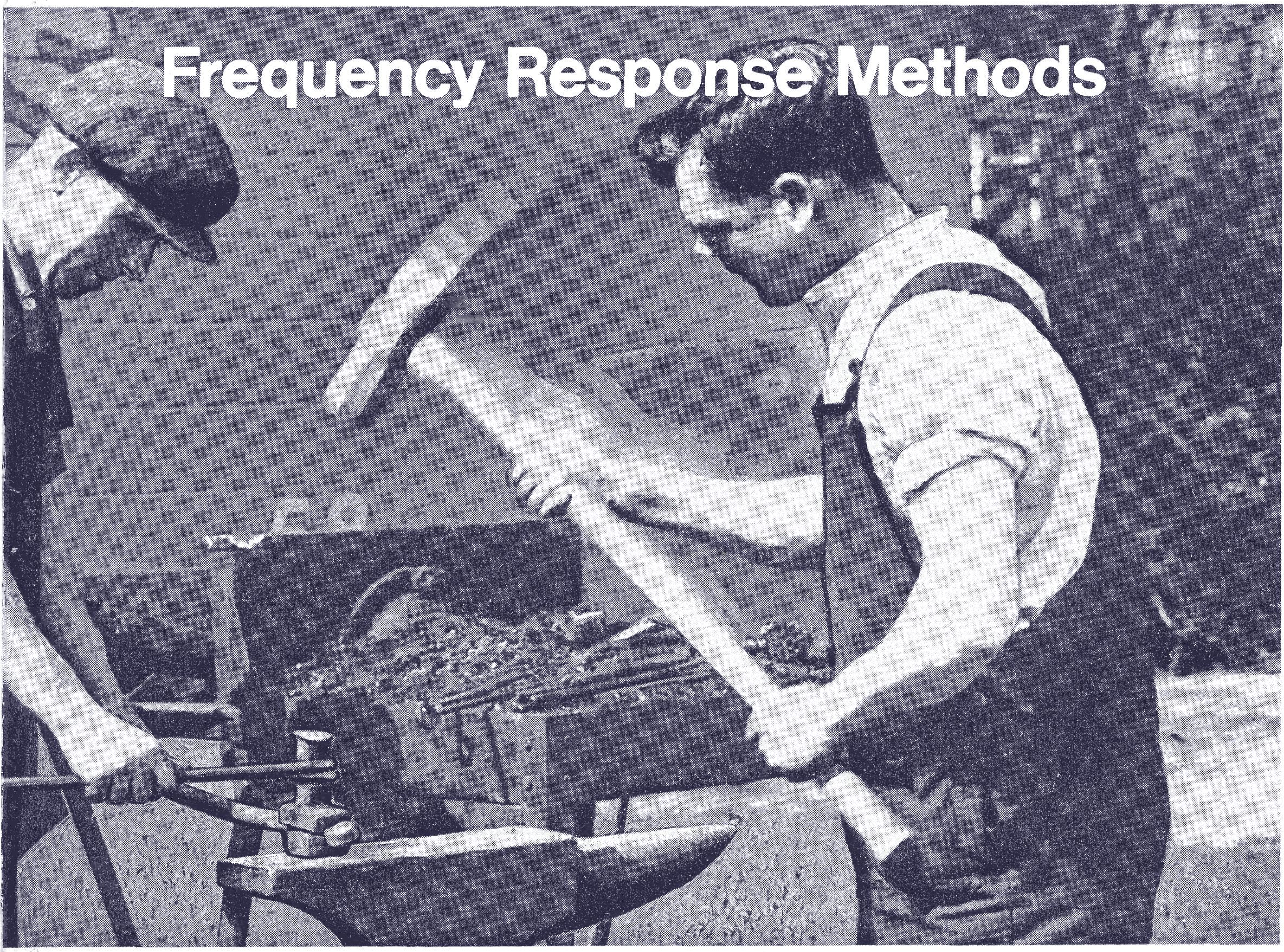
Frequency Response Methods



Brüel & Kjær

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TECHNICAL REVIEW

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Contents

On the Measurement of Frequency Response Functions

by

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ABSTRACT

After a brief introduction to frequency response measurements the three basic methods presently in use, i.e. the frequency sweep method, the impulse response transformation method, and the wide band random noise method are discussed in some details with major emphasis on dynamic range and measurement time considerations.

It is shown that a surprisingly accurate, (approximated) formula can be derived which governs the maximum frequency sweep rate for steady state measurement conditions. The formula expresses the fact that the (maximum) sweep rate, S (in Hz/s), should be of the order of the test device resonant bandwidth, Δf, squared. The application of this formula to various practical frequency sweep systems is illustrated.

Regarding the impulse response transformation method it is shown that a considerable loss in dynamic measurement range occurs when this method is used. On the other hand, when combined with what is commonly termed "real time analysis" this is the fastest frequency response measurement method in existence.

Although the wideband random noise method may be the most generally applicable method in practice it is found that not only does the method suffer from loss in measurement dynamic range, but to obtain reliable statistical accuracies in the results, also relatively long measurement times are required.

Finally, the pro's and con's of the different methods are briefly reviewed.

SOMMAIRE

Après avoir brièvement donné une introduction aux mesures des réponses en fréquence, cet article présente et discute en détails les trois méthodes fondamentales employées aujourd'hui: la méthode à balayage de fréquence, la méthode de transformation de la réponse impulsionnelle et la méthode à bande large de bruit aléatoire. L'accent est mis sur les considérations de gamme dynamique et de durée des mesures.

Il y est montré qu'une formule d'une précision surprenante, obtenue par approximations peut être dérivée, qui gouverne la rapidité maximale du balayage en fréquence pour des conditions de mesure bien définies. La formule exprime le fait que la rapidité (maximale) du balayage: S (en Hz/s), doit être de l'ordre du carré de la largeur de bande, Δf, de la réso-

nance du système étudié. On a illustré l'application de cette formule à l'aide de différents systèmes pratiques de balayage en fréquence.

Il apparaît qu'une perte considérable de gamme dynamique de mesure survient avec la méthode de transformation de la réponse impulsionnelle. Par contre, quand cette méthode fait appel aux moyens de ce que l'on appelle: "Analyse en temps réel", on a le système de mesure de la réponse en fréquence le plus rapide qui soit.

Bien que la méthode à bande large de bruit aléatoire soit la méthode la plus généralement applicable en pratique, on a trouvé que non seulement elle souffre d'une perte de gamme dynamique de mesure, mais aussi que l'obtention de résultats ayant des précisions statistiques satisfaisantes, demande de longues durées de mesure.

Enfin, on a passé brièvement en revue le pour et le contre des différentes méthodes.

ZUSAMMENFASSUNG

Nach einer kurzen Einführung über Frequenzgangmessungen, werden die drei z.Z. üblichen Methoden, nähmlich die Messung mit Sinus gleitender Frequenz, mit Impulsen und Fourier-Transformation sowie mit Breitbandrauschen näher besprochen, wobei das Hauptaugenmerk auf Dynamikbereich und Meßdauer liegt.

Es wird gezeigt, daß sich eine überraschend genaue Näherungsgleichung ableiten läßt, welche die maximale Frequenzdurchlaufgeschwindigkeit unter konstanten Meßbedingungen angibt. Die Gleichung besagt, daß die Durchlaufgeschwindigkeit S (in Hz/s) höchstens gleich dem Quadrat der Resonanzbandbreite Δf des zu prüfenden Objekts sein soll. Die Anwendung dieser Gleichung bei verschiedenen Meßanordnungen mit Sinus gleitender Frequenz in der Praxis wird illustriert.

Bezüglich der Impulsmethode mit Fourier-Transformation wird aufgezeigt, daß dabei ein beträchtlicher Verlust im Dynamikbereich auftritt. Wenn diese Methode allerdings in "Echtzeit" ausgeführt wird, ist dies der derzeitig schnellste Weg zur Messung von Frequenzgängen.

Obwohl auch die Messung mit Breitbandrauschen eine allgemein anwendbare Methode sein könnte, hat sich in der Praxis herausgestellt, daß diese Methode nicht nur mit einem Verlust an Dynamikbereich behaftet ist, sondern daß auch relativ lange Meßzeiten erforderlich sind, um eine ausreichende statistische Sicherheit der Meßresultate zu erreichen.

Zum Schluß werden alle Pro und Contras der verschiedenen Meßmethoden kurz wiederholt.

Introduction

Frequency response functions may be presented graphically in various ways. They may be presented in terms of the real and the imaginary parts of the complex response functions, in terms of the modulus and phase functions, or in terms of vector diagrams in polar coordinates. In all types of presentation frequency is the independent variable.

As phase-information is (normally) of less practical value to the engineer than modulus-information this paper more or less concentrates itself on the measurement of frequency response function modulii. To measure this function several methods are available in practice.

Historically the first (and still used) electrical method consisted of applying a sinusoidal voltage of a particular frequency and amplitude to the system under test, and then measuring the steady state response of the system to this particular signal on a voltmeter. By noting down on a graph paper the ratio Vout/Vin and changing the input signal frequency a complete Vout/Vin (and thus modulus) curve could be obtained as a function of frequency. This is, however, a rather time-consuming method of measurement, and other methods have been sought and found.

One such method is the automatic recording of Vout/Vin — ratios in terms of dB (decibels) on preprinted, frequency calibrated graph paper, using the mechanical drive of a level recorder to synchronize the paper with the frequency sweep of the input signal generator.

Other fully electronically controlled sweep systems performing the same operation are also at present in common usage.

The advent of the digital computer and so-called real time frequency analyzers, made still other methods of frequency response measurements practically available, such as the method of "real time" Fourier transformation of unit impulse responses. Here an approximated unit impulse is applied to the input of the system under test, while the output time function response is transformed, via some sort of Fourier transformation technique, to the system's frequency response function. *This method is the fastest measurement method in existence.* As will be shown later in the text some practical disadvantages are, however, connected with the impulse testing techniques.

While the frequency sweep systems, basically, consider the response of the test-object to **one** particular frequency at a time (series approach), the impulse test technique considers the response of the object to a, theoretically infinite, number of frequencies simultaneously (parallel approach).

Another measurement method of the parallel type consists of exciting the test-object by means of wide-band random noise and performing a Fourier (spectrum) analysis, either series or parallel, of the output signal. Although several variants of the above mentioned techniques are possible in practice three "basic" measurement methods remain:

1. The frequency sweep method.

2. The impulse response transformation method.

3. The wide band random noise method.

The pros and cons of the methods will be discussed in this paper, both in general terms, and as applied to some typical measurement situations.

The Frequency Sweep Method

The frequency sweep method discussed in the following is based on sweep speeds of the order of magnitude which allows the existence of steady state measurement conditions to be assumed. (If the sweep speed becomes very high the actual measurement conditions tend towards those existing under impulse excitation of the device being investigated).

Typical measurement arrangements used in frequency sweep systems are shown in Fig. 1, where the sweep can be either mechanically controlled from a motor in the level recorder, or it can be controlled from an external electronic unit (which then also controls the paper drive system in the recorder).

Before discussing the application of the arrangements to practical measurement situations, some general features should be briefly outlined.

The technically most important features of a measurement system are the dynamic range and frequency resolution available, as well as the total measurement time involved. As only one sinusoidal signal component is applied to the system under test at a time a maximum dynamic measurement range can be obtained, and, in principle, also an infinitely high frequency resolution.

However, it could be argued that because only one signal frequency component is applied to the test-system at a time, the total measurement time required for recording a complete frequency response characteristic may be relatively long. This is, to a certain extent, true, but depends greatly upon the characteristics of the measurement system as well as those of the device being tested. On the other hand, the measurement equipment itself is of the least expensive type of to-day's instrumentation facilities.

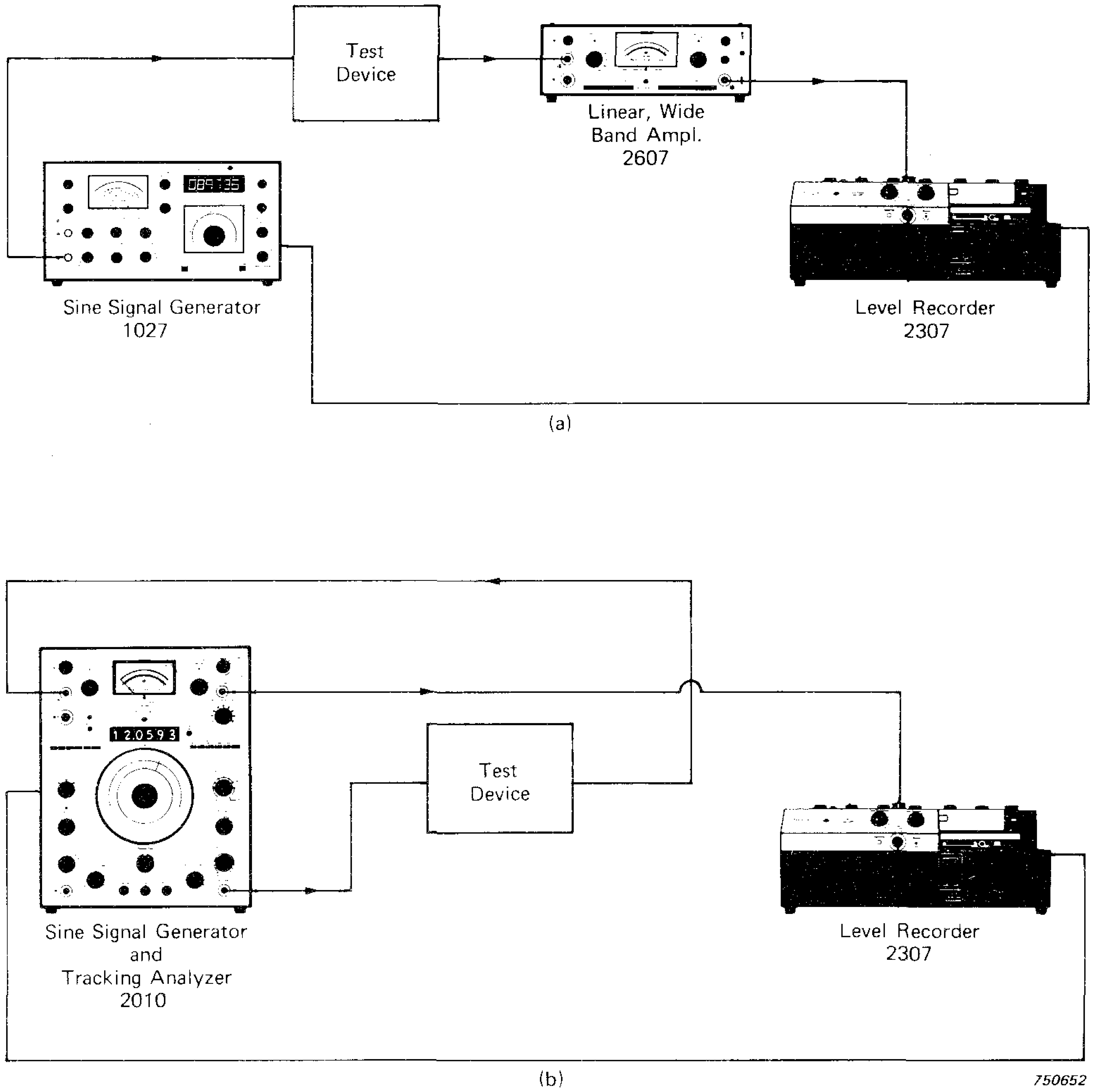

Fig. 1. Typical frequency sweep test systems with
a) linear amplification of the output from the test device, and
b) selective amplification of the output

Even though the dynamic range available for measurements in an arrangement, such as the one shown in Fig. 1a, is quite large, it may be extended further by substituting the linear amplifier by a narrow band frequency selective instrument "tracking" the signal generator (Fig. 1b). The narrower the bandwidth of the frequency selective amplifier (analyzer) is, the larger becomes the available dynamic range of measurement. On the other hand, in practice there are certain limits as to how narrow the frequency band should be, a fact, which will be discussed later in this paper.

Returning now to the question of measurement time involved in frequency sweep tests this is, in general, related to the maximum allowable sweep speed. The maximum allowable sweep speed again depends upon the physical characteristics of the system being investigated and the accuracy required from the measurements.

Considering first a measurement system of the type shown in Fig. 1a — i. e. a system, where the output from the device being investigated is fed to a linear, wide-band amplifier — the maximum allowable frequency sweep speed is limited by: 1) the physical characteristics of the test device itself, and: 2) the detector circuit of the measurement system.

Starting with 1) above, assume that the device under test contains a number of resonances. The frequency response modulus curve for such a device will then show a number of resonance "peaks" and "valleys", as exemplified in Fig. 2. When this kind of measurement situation is to be analyzed a most important consideration to be made is how a sweeping sinusoidal input signal affects the build-up and decay of a single resonance response.

If the input signal has a constant maximum amplitude and is swept very slowly through the frequency range covered by the resonance, the amplitude of the output signal will follow the resonance steady-state response curve very closely. However, the faster the frequency sweep is the more the response will deviate from the steady-state response, quite apart from the fact that the sweeping signal may "beat" with the resonant decay.

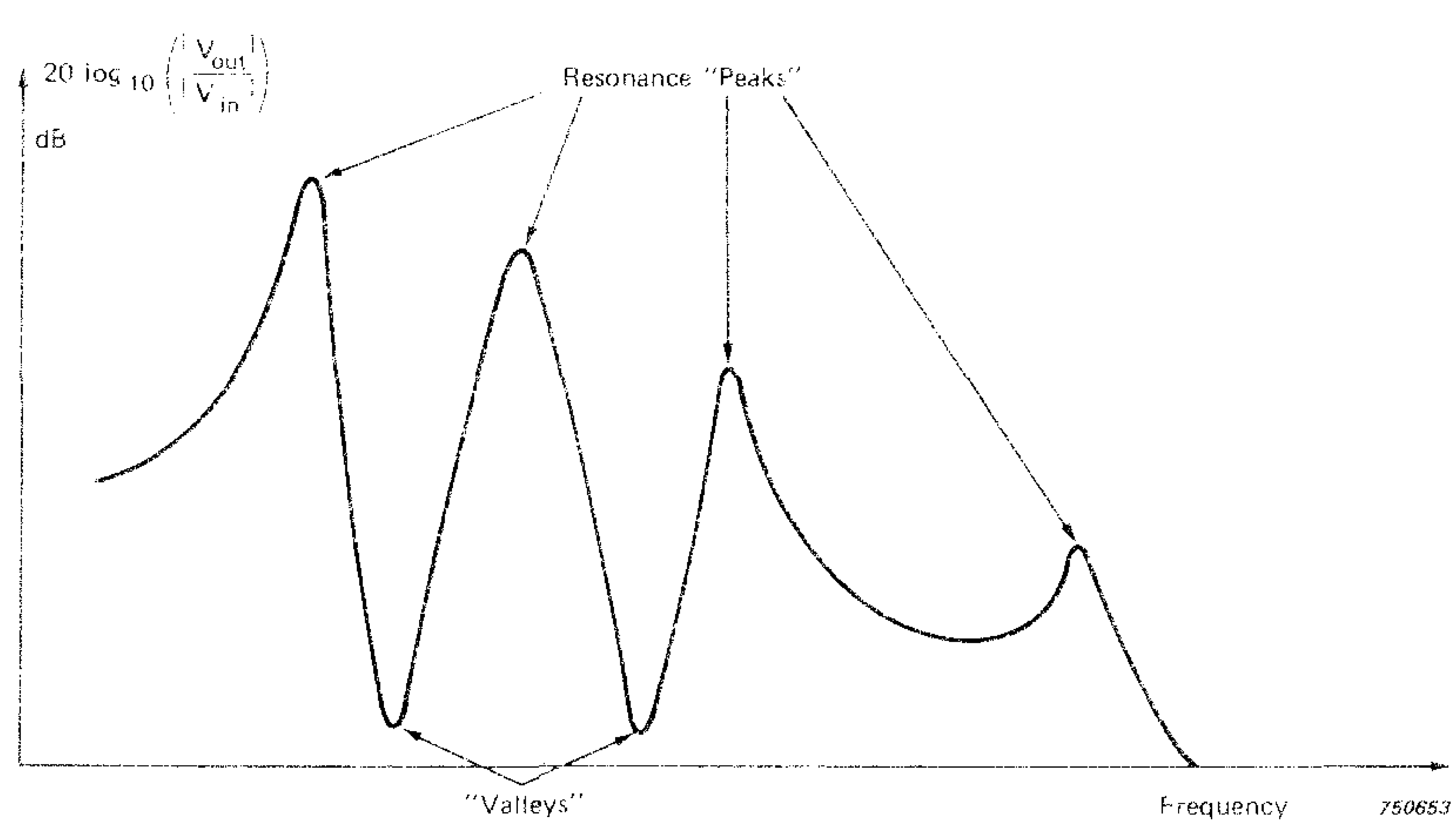

Fig. 2. Typical frequency response modulus curve for a multiresonant test device

Assuming, for the sake of simplicity, that the frequency sweep function is linear in the frequency region around the resonance "peak" the input signal may within this range be written in the form:

$$X_{in}(t) = X_0 \sin(at^2 + bt)$$

where $2at + b = \omega(t)$, $d\omega/dt = 2a$, and $b = \omega_L$ = low frequency limit for the above frequency range assumption. The time function response of the resonance to this sweep condition can be obtained by convolving the input time function with the unit impulse response of the resonance. For resonances with reasonably high Q-values* the unit impulse response is given by:

$$h(t) = X_{peak} \exp\left(-\frac{\omega_0}{2Q} t\right) \sin(\omega_0 t + \varphi)$$

where X_{peak} is the maximum response amplitude, $\omega_o = 2\pi f_o = 2\pi$ times the resonance frequency, $Q = f_o/\Delta f$ and φ is a phase angle, the magnitude of which depends upon the actual configuration of the resonance system.

With the above assumption in mind, the convolution can be mathematically formulated as:

$$X_{out}(t) = \int_{t_1}^{t} X_0 \sin(a\tau^2 + b\tau)\, X_{peak} \exp\left[-\frac{\omega_0}{2Q} x (t-\tau)\right] \sin[\varphi + \omega_0(t-\tau)]\, d\tau$$

Here t_1 corresponds to the instant in time where the instantaneous sweep frequency $f_L = \omega_L/2\pi$ occurs.

By looking a little closer at the above expression it is readily seen that the mathematics involved is not easily handled, even by present day's numerical computation methods.

To obtain a somewhat simplified mathematical model, which would allow reasonable sweep speed estimates to be made in practice consider the following:

As the response time of the resonance to external excitations is determined by the response *envelope,* and as the phase difference between

* The Q-value of a resonance may be expressed as $Q = f_0/\Delta f$ where f_0 is the resonance frequency and Δf is the half power bandwidth of the resonance. This expression is reasonably accurate, when $Q > 5$

$\sin(a\tau^2 + b\tau)$ and $\sin[\varphi + \omega_o(t - \tau)]$ varies violently with τ, it is assumed that a reasonable approximation to the exact convolution may be obtained by considering the "energies" involved, i. e. by convolving the square of the response *envelope* with the square of the excitation envelope. The above exact expression for the instantaneous output signal value $X_{out}(t)$ can then be transformed into an "energy-envelope" relationship of the kind:

$$X^2_{out}(t) = c\int_{-\infty}^{t} \frac{\exp\left[-\frac{\omega_0}{Q}(t-\tau)\right]}{1+(\tau/\tau_0)^2}\, d(\tau/\tau_0)$$

where c is a normalizing constant and τ_0 characterizes the excitation signal envelope in that:

$$\tau_0 = \frac{\Delta f}{2S}$$

see also Fig.3.

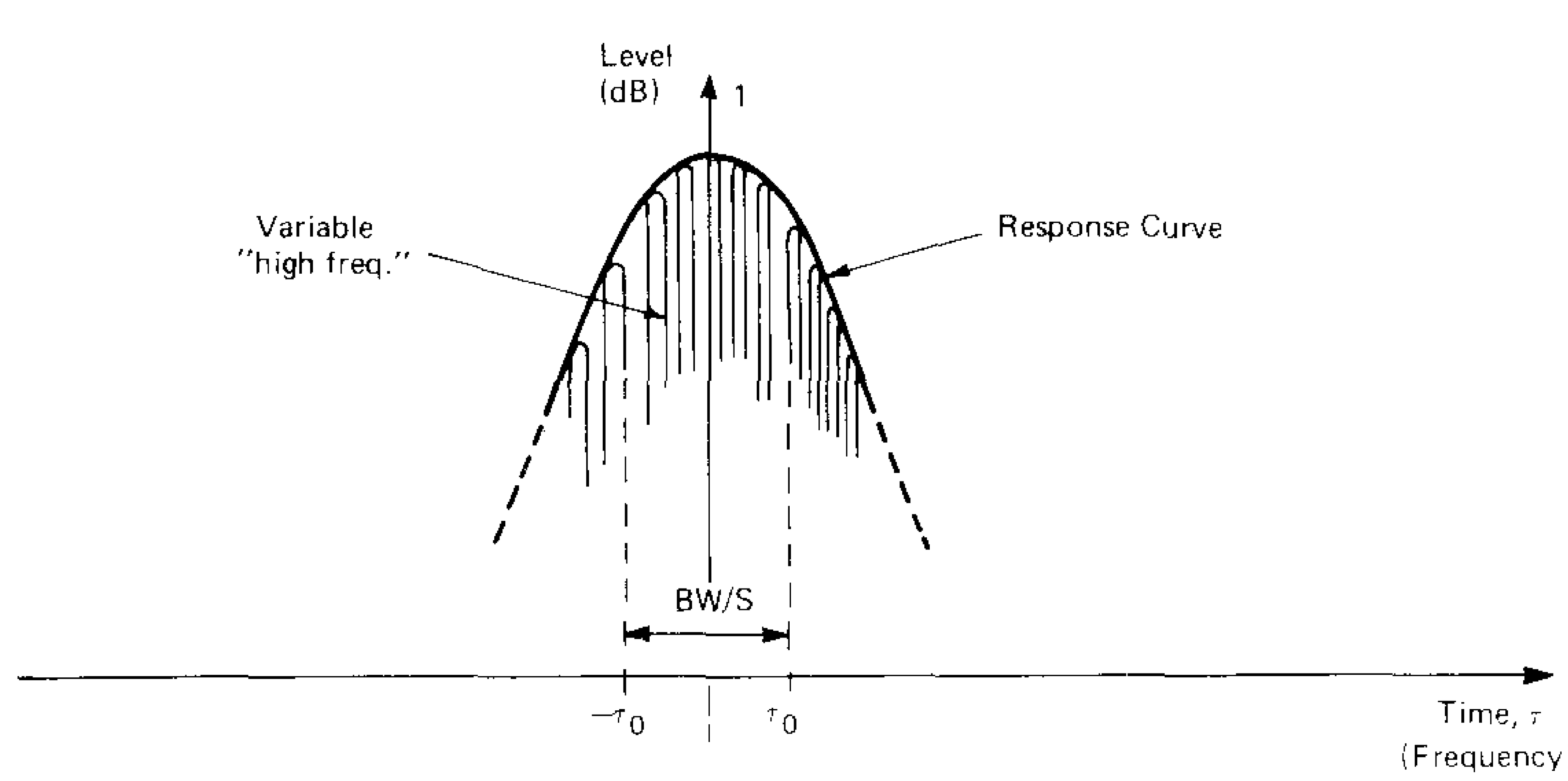

Fig.3. Illustration of a resonance "envelope-pulse". Resonance bandwidth BW = $\Delta f = 2\tau_0 \times S$, where S is the frequency sweep speed

Even this integral is quite complicated to solve and use had to be made of numerical techniques and digital computations. The result is shown in Fig.4, and indicates that:

$$\left.\begin{array}{l}\text{Error in Maximum}\\ \text{Resonance Response}\end{array}\right\} = F_1\left(\frac{S}{(\Delta f)^2}\right)$$

where $S = df/dt$ = frequency sweep speed.

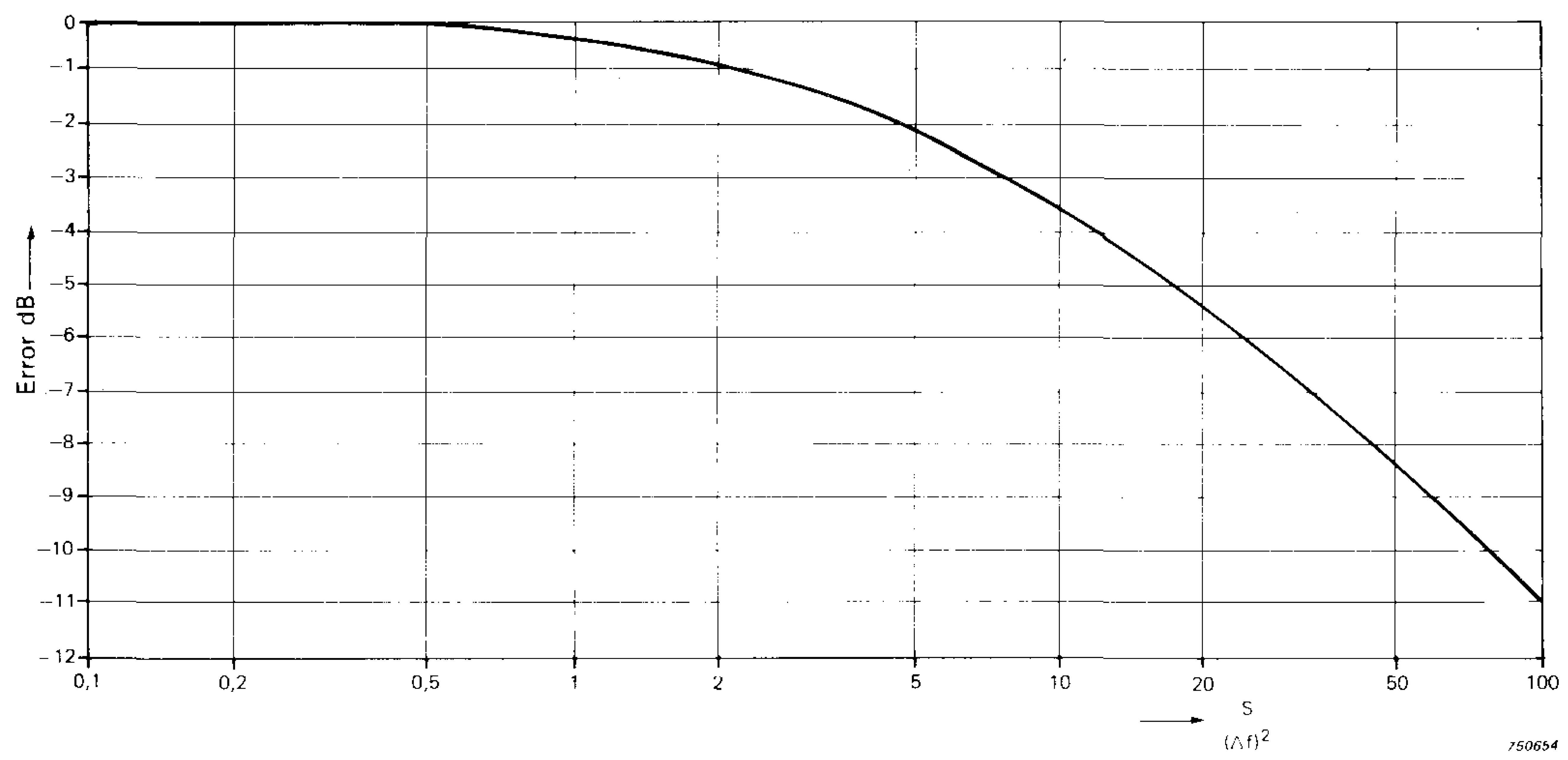

Fig. 4. "Error" in response maximum as a function of ($S/\Delta f^2$)

To keep the error in the maximum resonance response in the order of a few tenth of a decibel then $S/(\Delta f)^2$ should be of the order of 1 or less, i. e. the following important expression can be formulated for the frequency sweep speed:

$$S = \frac{df}{dt} \leqslant (\Delta f)^2$$

This formula has been checked experimentally and found surprisingly accurate, considering the approximations used in its derivation.*

Remembering now the "definition" of resonance Q-values:

$$Q = \frac{f}{\Delta f}$$

then $$\Delta f = \frac{f}{Q} \text{ and } (\Delta f)^2 = \frac{f^2}{Q^2}$$

whereby the sweep speed formula takes the form:

$$\frac{df}{dt} \leqslant \frac{f^2}{Q^2}$$

If the device being investigated consists of a number of resonances with constant Q-values then the maximum allowable frequency sweep speed is given by:

---

* The formula actually corresponds to the "famous" relationship: $\Delta f\ \Delta t \geqslant 1$ as $S = \Delta f/\Delta t \leqslant (\Delta f)^2$

$$\frac{df}{dt} = \frac{1}{Q^2} f^2$$

and the period of time necessary for one sweep from a lower frequency "limit" f_L to higher frequency "limit", f_H, becomes:

$$\int_0^T dt = \int_{f_L}^{f_H} Q^2 \frac{df}{f^2} = Q^2 \left(\frac{1}{f_L} - \frac{1}{f_H}\right)$$

or

$$T = \frac{Q^2}{f_L} \left(1 - \frac{f_L}{f_H}\right)$$

This frequency sweep function may be termed *hyperbolic,* for obvious reasons.

If the device under test contains resonances, the *Q-values of which increase with the square-root of frequency,* ($Q = C_x \sqrt{f}$), the corresponding maximum allowable sweep speed becomes:

$$\frac{df}{dt} = \frac{f^2}{Q^2} = \frac{1}{C_x{}^2} f$$

and the total sweep time:

$$T = C_x{}^2 \ln\left(\frac{f_H}{f_L}\right)$$

The frequency sweep function governing this type of sweep is commonly termed *logarithmic.*

Should the device contain resonances the *Q-values of which increase linearly with frequency,* i. e. $Q = C_y \times f$, then the maximum allowable sweep speed must be:

$$\frac{df}{dt} = \frac{f^2}{Q^2} = \frac{1}{C_y{}^2}$$

which results in a sweep time of:

$$T = C_y{}^2 (f_H - f_L)$$

i. e. the corresponding frequency sweep function becomes *linear.*

It may be of considerable interest at this stage to summarize the above obtained results for the purpose of comparison. This has been done in the Table 1, and the minimum sweep times necessary to cover the frequency range 20 Hz to 20000 Hz under the various conditions are illustrated in Fig.5.

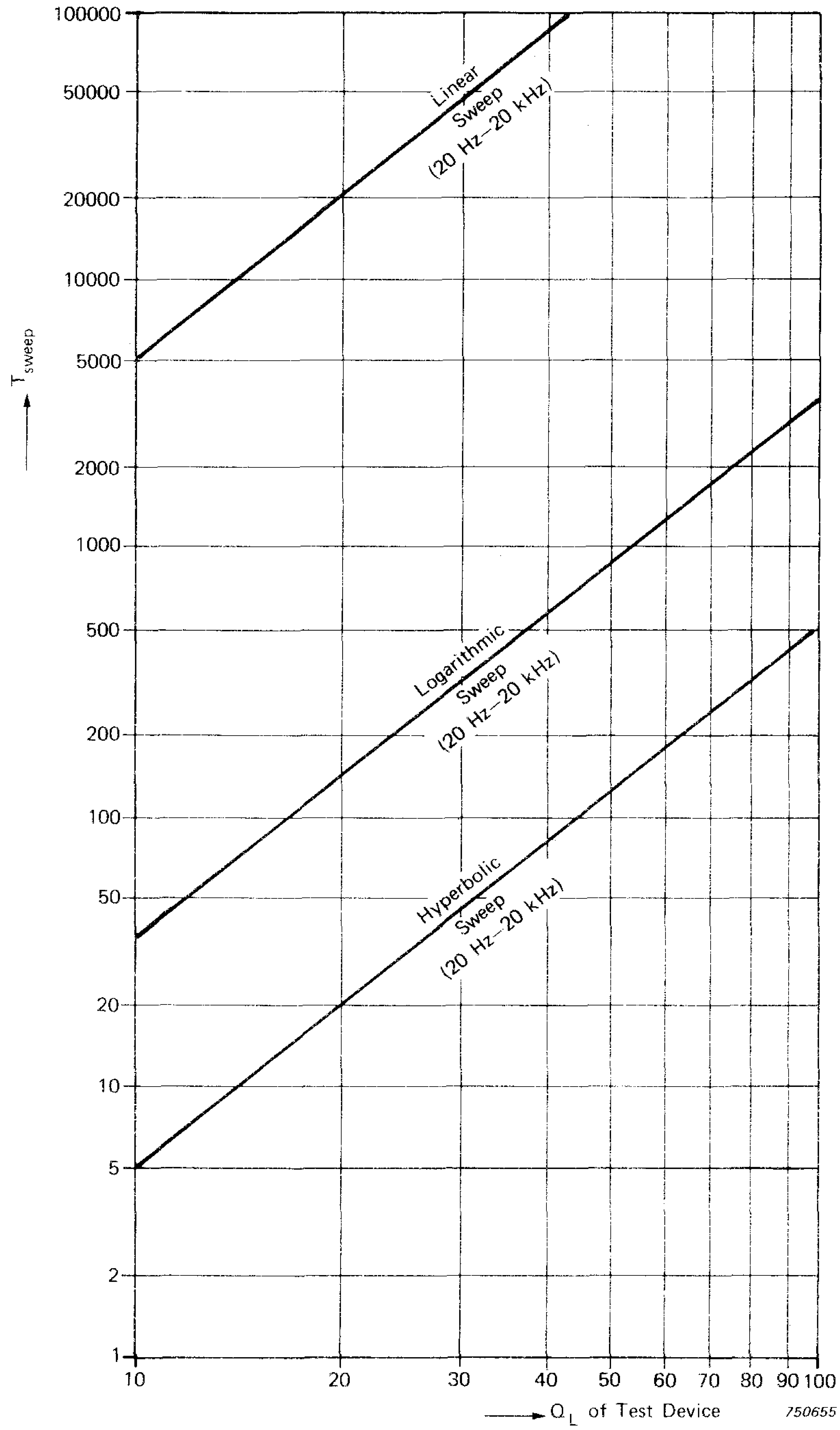

Fig.5. Chart showing the "minimum" frequency sweep times necessary to cover the frequency range 20 Hz to 20 kHz as a function of the test device Q-factors (or roll-offs)

Frequency Function	Resonance Char. of Device under Test	Frequency Sweep Time
Hyperbolic	Constant Q	$T = Q^2\left(\frac{1}{f_L} - \frac{1}{f_H}\right) = \frac{Q^2}{f_L}\left(1 - \frac{f_L}{f_H}\right)$
Logarithmic	$Q = C_x \times \sqrt{f}$	$T = C_x{}^2 \ln\left(\frac{f_H}{f_L}\right) = \frac{Q_L{}^2}{f_L} \ln\left(\frac{f_H}{f_L}\right)$
Linear	$Q = C_y \times f$	$T = C_y{}^2(f_H - f_L) = \frac{Q_L{}^2}{f_L}\left(\frac{f_H}{f_L} - 1\right)$

Table 1.

Note that the constants C_x and C_y have here been chosen in such a manner that the sweep speed at the lower "limit" frequency is the same for all three frequency sweep functions, and depends only upon the expected Q-value of the resonances in the test item *at that particular Frequency. This choice of constants may, however, not always be the most appropriate one in practice.*

An interesting feature of the measurement system, which becomes clear from looking at the mathematical expressions for the frequency sweep speed, as well as from looking at Fig.5, is that when Q decreases the maximum allowable sweep speed increases. Or, in other words, when the device under test does **not** contain any resonances, but shows a practically "flat" frequency response curve infinitely high frequency sweep speeds could, theoretically, be employed.

However, all realizable physical devices produce frequency response characteristics which are "limited" at higher frequencies. The "limitation" may be caused either by more or less complicated resonant phenomena, or the response curve may show a "simple" frequency "roll-off", see Fig.6.

In practice, therefore, the maximum allowable frequency sweep speed is governed by the fact that the measurement system must be capable of reproducing such as a "roll-off", even if the response characteristic of the device being tested is flat up to the start of the "roll-off", Fig.6b.

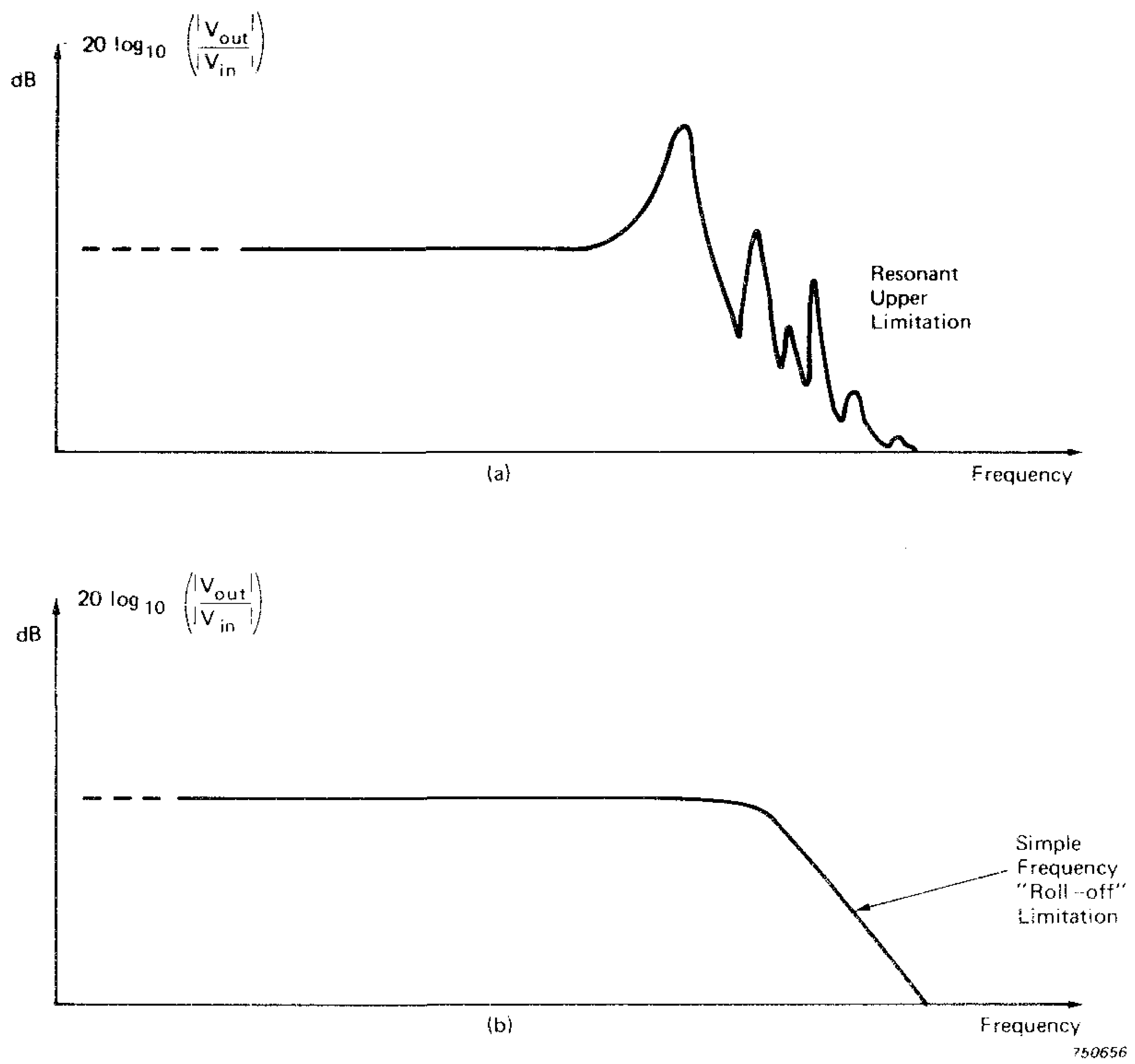

Fig. 6. Sketch illustrating common high-frequency limitations imposed upon physical systems by
a) complex resonant phenomena
b) simple frequency roll-off

So far only the physical characteristics of the device being investigated have been considered in the derivation of maximum allowable frequency sweep speeds. As mentioned earlier, however, also the measurement system itself, in particular the detector circuit in the wide-band amplifier (Fig. 1a), imposes practical limitations upon the derived frequency sweep formulae.

The detector circuit employed in commonly available measurement systems may be either of a type detecting an instantaneous signal value, normally the peak value, or of a type detecting some form of time-averaged value, for instance the RMS (root mean square) value.

Although signal peak detectors can be designed in various ways for different measurement purposes the fact remains that this kind of detector circuit allows the fastest form of signal level detection to be made, as the detection depends upon an *instantaneous* signal value only.

A "fast" signal peak detector could, for instance, be designed in such a

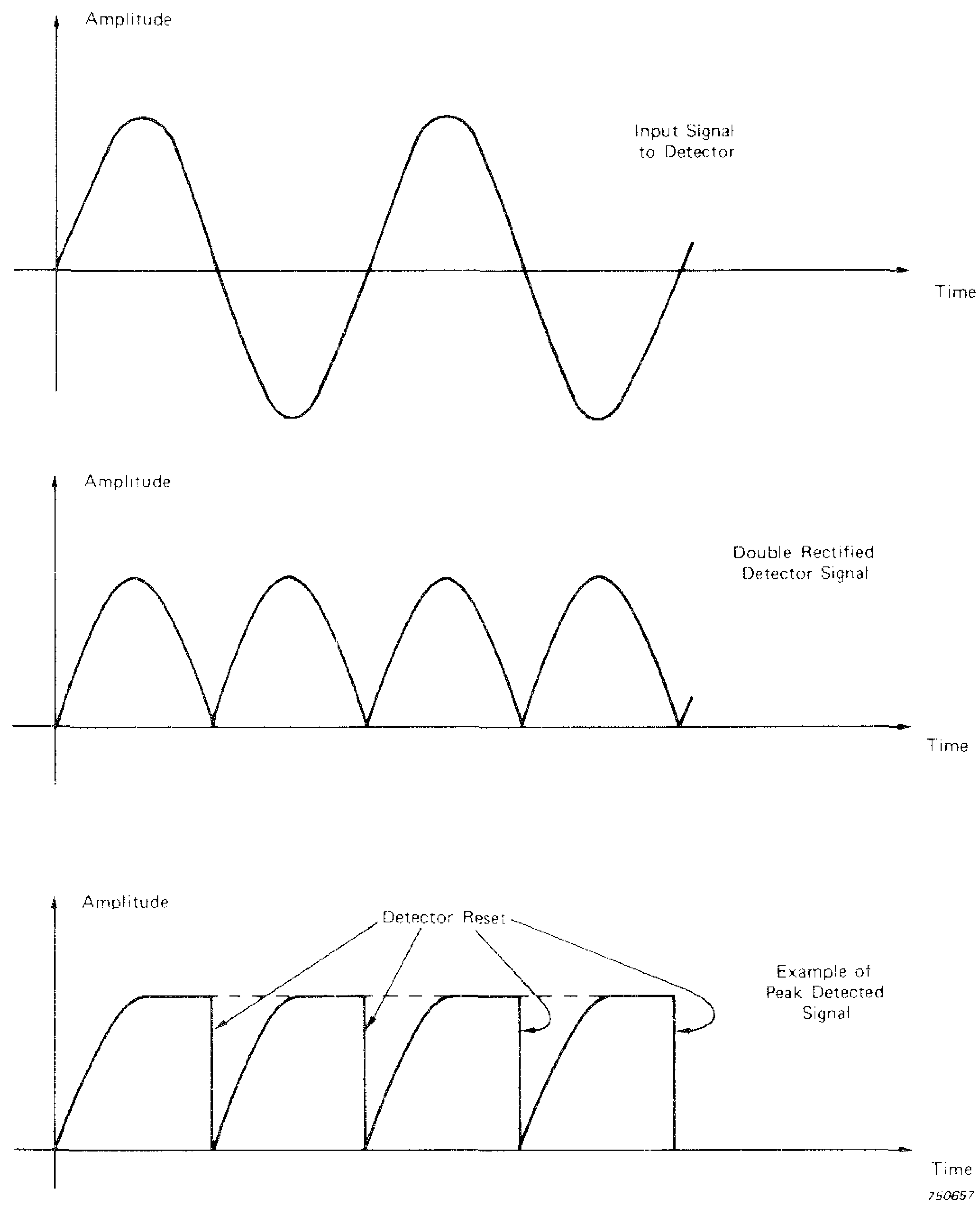

Fig. 7. Example of instantaneous (peak) value detection of a sine-wave signal

manner that just after the detection of a signal peak value the detector is reset to feel the next peak. If a double rectifier system is used it is thus possible to detect two signal peak values per period of the signal, the positive signal peak and the negative signal peak, Fig.7, whereby even very fast signal level changes can be detected.

Generally speaking, a detector of this type would pose no "extra" restriction upon the maximum allowable sweep speed to be used in frequency sweep measurements, and the sweep formulae developed above would remain valid. If, on the other hand, some form of time-averaging detector system is used the averaging process may, at "high" sweep speeds, affect the system performance. "High" sweep speeds can be used when the device being investigated contains resonances with low Q-values, or moderately steep frequency roll-offs.

For resonance Q-values less than 10 the sweep time formulae derived earlier in this section must be modified as indicated in Table 2 (see also Appendix A):

Frequency Sweep Function	Resonance Charact. of Device under Test	Averaging Time	Frequency Sweep Time
Hyperbolic	Constant Q	Variable $T_{av} = \frac{3}{f}$	$T = \frac{10 \times Q}{f_L}\left(1 - \frac{f_L}{f_H}\right)$
Logarithmic	$Q = C_x \times \sqrt{f}$	Variable $T_{av} = \frac{3}{\sqrt{f_L \times f}}$	$T = \frac{10 \times C_x}{f_L} \ln\left(\frac{f_H}{f_L}\right)$ $T = \frac{10 \times Q_L}{f_L} \ln\left(\frac{f_H}{f_L}\right)$
Linear	$Q = C_y \times f$	Constant $T_{av} = \frac{3}{f_L}$	$T = 10 \times C_y (f_H - f_L)$ $T = \frac{10 \times Q_L}{f_L}\left(\frac{f_H}{f_L} - 1\right)$

Table 2.

Corresponding minimum sweep times required to cover the frequency range 20 Hz to 20000 Hz are illustrated in Fig.8 (see also Fig.5).

Returning now to Fig.1 it was stated that even though the dynamic measurement range of a measuring system such as the one shown in Fig.1a) is very large, it could be still improved if it was changed to perform so-called selective measurements, Fig.1b). It was also stated that the bandwidth of the tracking analyzer should be small, and that certain practical limits existed as to how narrow the band should be.

Considering the reasoning that lead to the derivation of the "basic" frequency sweep expression:

$$S \leqslant (\Delta f)^2$$

it is readily understood that the bandwidth of the tracking analyzer must be of the order of, or larger than, Δf. This can be easily verified

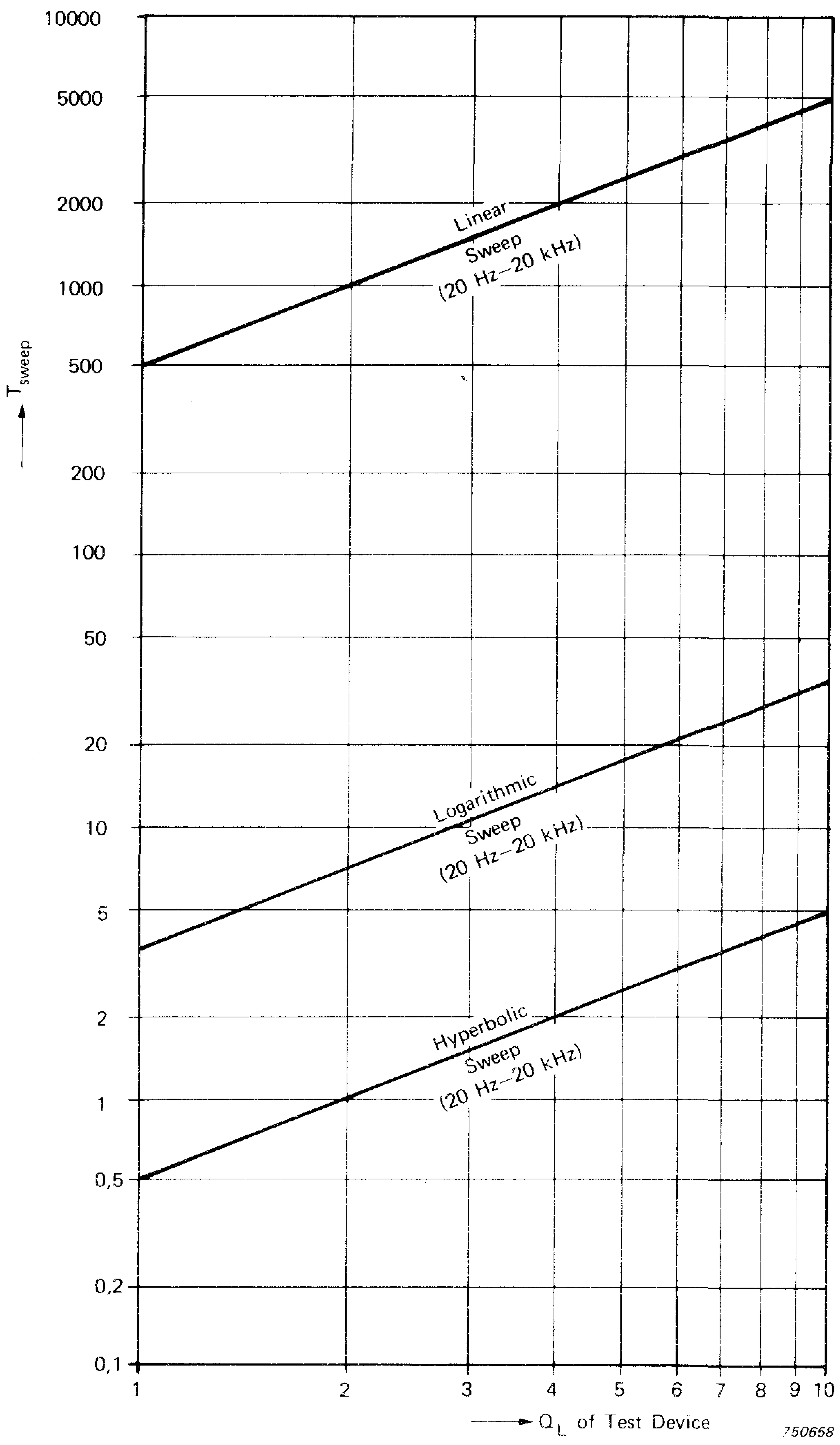

Fig. 8. Chart showing the "minimum" frequency sweep time necessary to cover the frequency range 20 Hz to 20 kHz when the detector system in the measurement arrangement is of the time-averaging (RMS) type, and the test device contains resonances with low Q-factors (or moderately steep frequency roll-offs)

experimentally, and a suitable condition for the analyzer bandwidth, BW, would be

$$BW_{Analyzer} \geqslant 3\ \Delta f$$

The gain obtained in dynamic range by including a tracking analyzer in the measurement system is, obviously, of the order of:

$$\text{Gain in Dynamic Range} = 10 \log_{10} \left[\frac{f_u - f_L}{BW_{Analyzer}} \right] \text{dB}$$

Although other measurement arrangements could be suggested for frequency sweep tests, including for instance a "compressor" circuit, the arrangements shown in Fig.1 may be considered basic for the purpose of this paper. The inclusion of a compressor loop may result in input signal distortion, unstability in the measurement set-up as well as other delay effects and should therefore be considered a separate subject on its own. Information related to the use of compressor circuits are given in the relevant instrument instruction manuals, as well as in the B & K Technical Reviews No. 4-1955, No. 2-1958, No. 4-1962 and No. 3-1965.

The Impulse Response Transformation Method

As mentioned in the introduction to the article this is the fastest frequency response measurement method in existence. It does, however, require a rather sophisticated and expensive measurement instrumentation. Furthermore, severe limitations with regard to dynamic range utilization exists, a fact which is discussed in some details below.

A typical measuring arrangement required for these kinds of measurements is sketched in Fig.9. The "Real Time Analyzer" indicated in the figure may consist of a hybrid type time-compression analyzer, such as the Brüel & Kjær Type 3348, a special FFT (Fast Fourier Transform) instrument, or a general purpose computer programmed for Fourier transformation operation together with the necessary A/D converter and preamplifier.

To allow for a somewhat more detailed discussion of 1) the dynamic range utilization of the instrumentation and 2) the time required for this kind of frequency response measurements consider first the case where the device being tested contains *one* resonant system only.

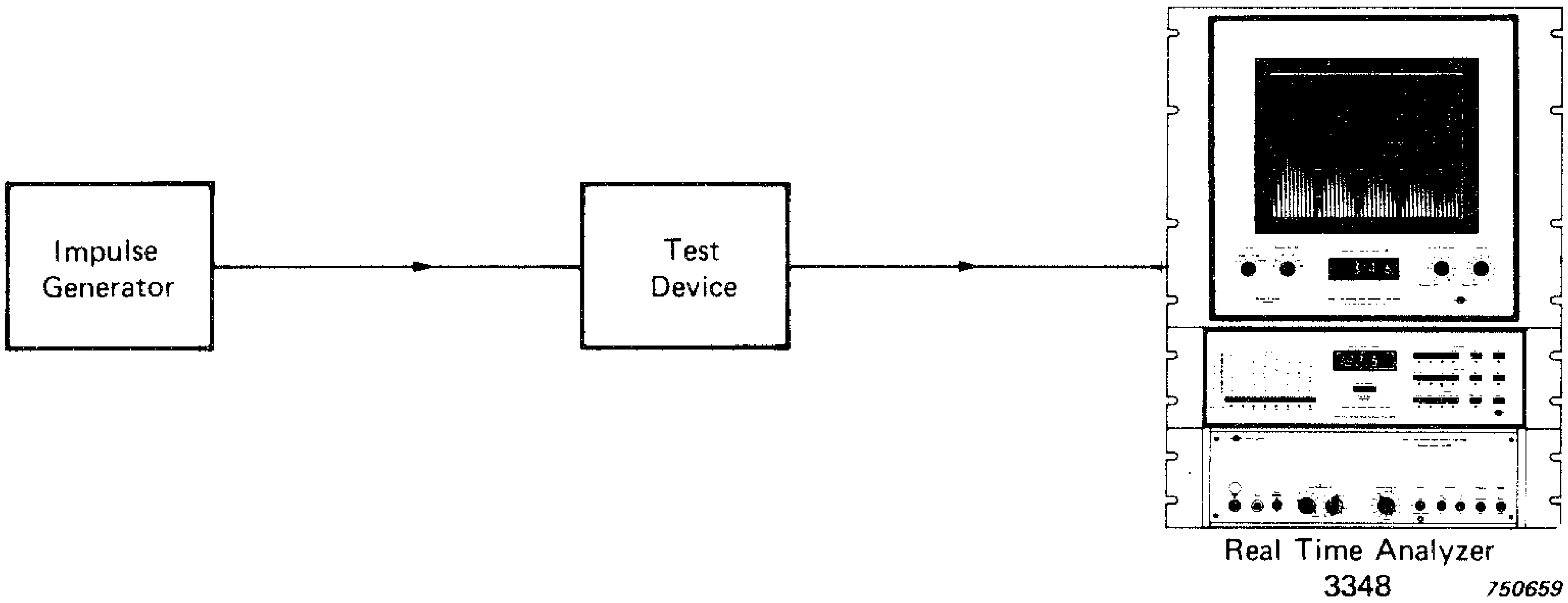

Fig. 9. Typical "real time" measuring arrangement used in the impulse response transformation test method

If a very short duration impulse (theoretically a δ-impulse) is applied to the test-device the time-function output from the (single degree of freedom) system may have the form:*

$$X_{out}(t) = X_{peak} \exp\left(-\frac{\omega_0}{2Q} t\right)\left[\cos(\omega_0 t) - \frac{1}{2Q} \sin(\omega_0 t)\right]$$

This is an oscillating transient as sketched in Fig. 10, and if $Q \gg 1$ then:

$$X_{out}(t) = X_{peak} \exp\left(-\frac{\omega_0}{2Q} t\right) \cos(\omega_0 t)$$

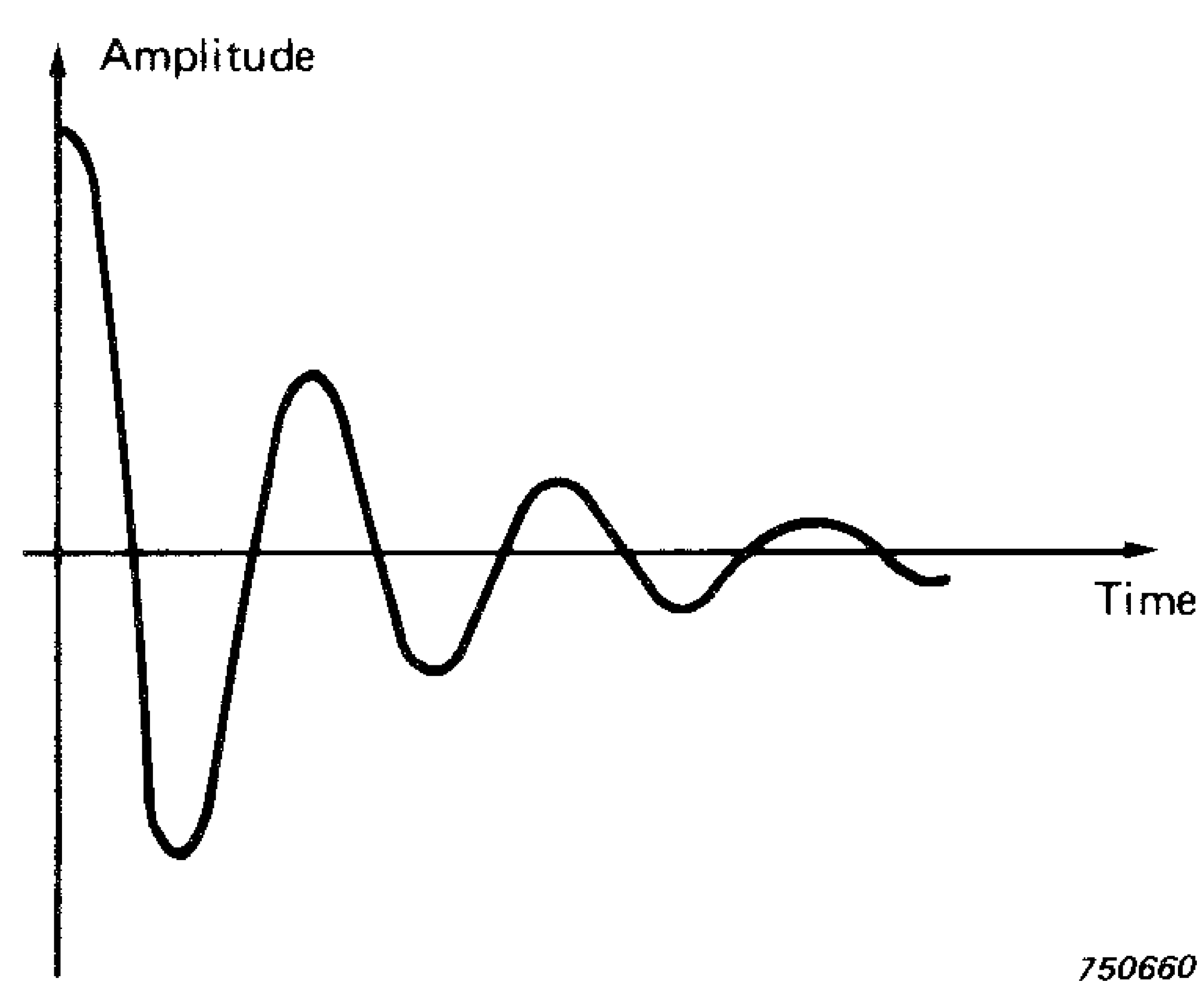

Fig. 10. Example of an oscillating transient time response function

*) The mathematical expression for the impulse response depends upon the actual physical configuration of the resonant system

To Fourier analyze the function $X_{out}(t)$ in practice the transient is normally captured in the memory of the analyzer, continuously recirculated, and analyzed as if it was a stationary periodic signal with a waveform equal to the transient waveform. If the recirculation frequency is $\Delta f_M = 1/T_M$ then the maximum value of the "transform" is (see Appendix B):

$$F(f)_{max} = F(f_0) = \frac{2X_{peak}}{T_M \ \Delta\omega} = 2X_{peak}\frac{\Delta f_M}{\Delta\omega} = \frac{1}{\pi} X_{peak}\frac{\Delta f_M}{\Delta f}$$

This expression actually gives the relationship between the maximum value of the time-function input signal *to* the real time analyzer, and the resulting maximum output *from* the analyzer for the case of a single degree-of-freedom test device, see Fig.11.

Assuming that the noise level at the input to the analyzer is the same, no matter what type of input signal is analyzed, the expression found above for $F(f_0)$ can be used directly in a discussion of the dynamic range available for the analysis of transient signals in practice.

If an analyzer frequency resolution of say $3\,\Delta f_M = \Delta f$, $(\Delta f = \Delta\omega/2\pi)$ is considered sufficient (error in $F(f_0)$ of the order of 0,5 dB or less) then:

$$F(f_0) = X_{peak}\frac{2\Delta f_M}{2\pi\ \Delta f} = \frac{X_{peak}}{3\pi}$$

Had the input signal to the analyzer been a stationary, harmonic signal of frequency $f_0 = \omega_0/2\pi$: $X = X_{s\,peak}\sin(\omega_0 t)$

then $\qquad F_{sine}(f_0) = X_{s\,peak} = X_{peak}$

This means that for a frequency resolution requirement of $\Delta f_M = \Delta f/3$ the difference in available dynamic range between a frequency sweep-

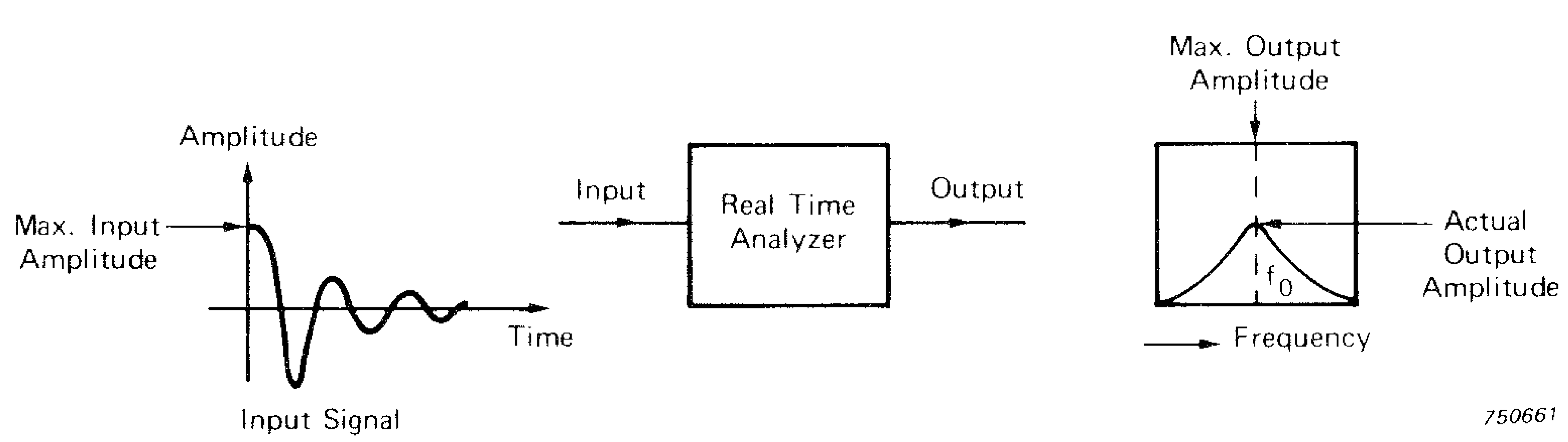

Fig.11. Sketch illustrating the "loss" in dynamic range during the frequency analysis of an oscillating transient

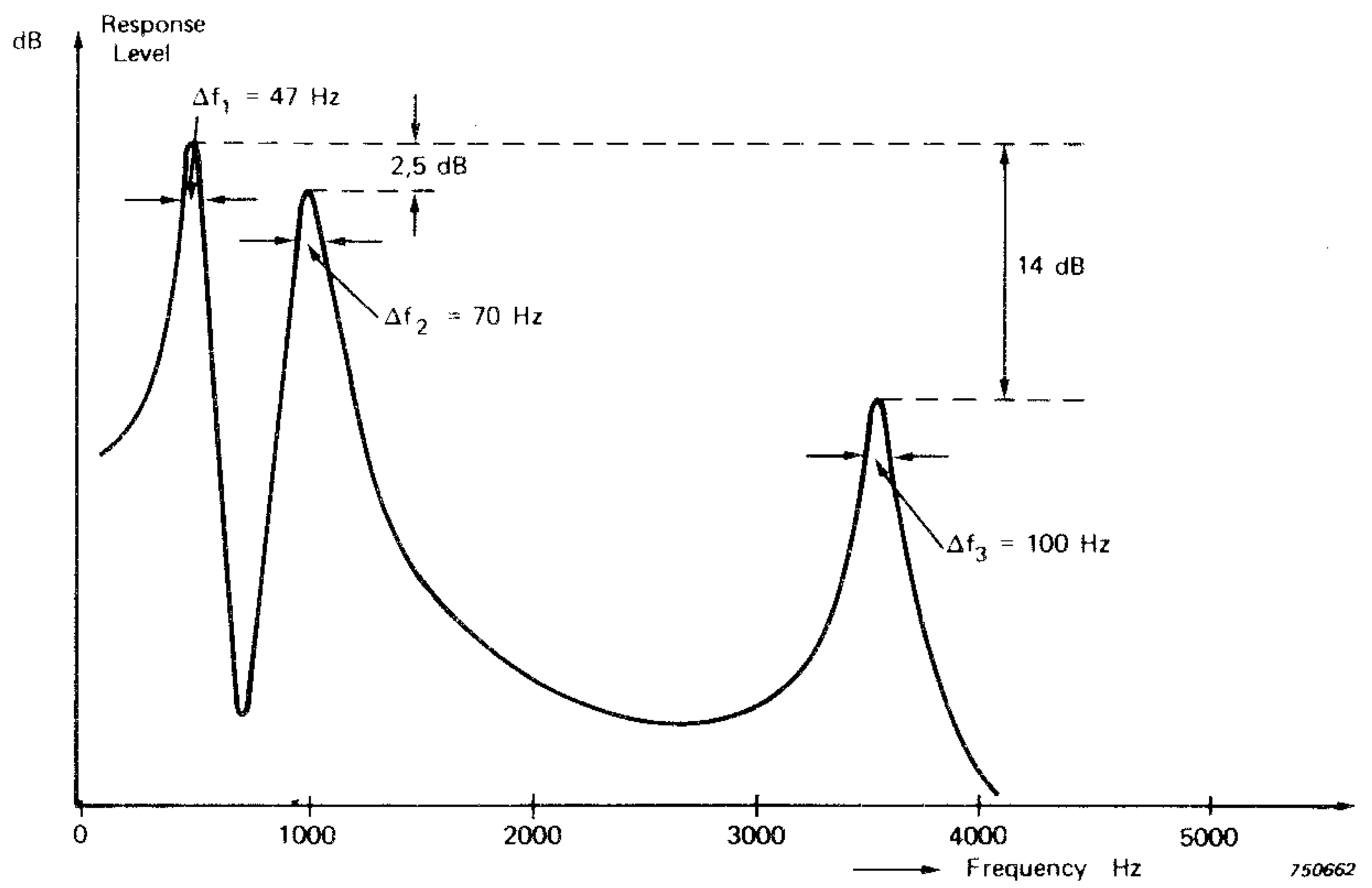

Fig. 12. Example of the frequency response function of a test device containing three resonances

measurement and an impulse response transformation measurement on a single degree-of-freedom test device is of the order of 3π (or, roughly, 20 dB).

If the device being investigated contains several resonances, such as indicated in Fig. 12, the dynamic range problem becomes very much more complicated, and *exact* relationships between frequency sweep measurements and impulse response transformation measurements can only be found by studying overall phase and frequency relationships. However, a rough estimate may be obtained in the following way:

Assume that each resonance spectral "peak" is approximated by a single degree-of-freedom response, and that (in the worst case) the total peak value of the time-function input signal to the analyzer consists of a linear summation of all the individual resonance X_{peak} -values, then:

$$X_{tot.peak} = \sum_n X_{npeak} = \frac{\pi}{\Delta f_M} \sum_n F(f_{0n})\,\Delta f_n$$

or

$$X_{tot.peak} = \pi \frac{\Delta f_1}{\Delta f_M} F(f_{01}) \left(1 + \sum_{n=2}^{n} \gamma_n\right) =$$

$$= X_{1peak} \left(1 + \sum_{n=2}^{n} \gamma_n\right)$$

where $\gamma_n = \dfrac{F(f_{0n})}{F(f_{01})} \dfrac{\Delta f_n}{\Delta f_1}$

See also Fig.12.

The "height" of each individual resonance peak, $F(f_{0n})$, can now be estimated:

$$F(f_{0n}) = \left(\frac{X_{n\,peak}}{X_{tot.peak}}\right) X_{S\,peak} \frac{\Delta f_M}{\Delta f_n}$$

where the maximum total peak, $X_{tot\,peak}$ as before, is equal to the maximum harmonic signal peak $X_{S\,peak}$.

This means that the "height" of each resonance "peak" in the frequency response function may be reduced by a factor of $X_{n\,peak} / X_{tot\,peak}$ *with respect to that which would have been obtained if the test device had contained the n'th resonance only.*

An example readily illustrates the above considerations:

Fig.13 shows the frequency response function of Fig.12 measured a) by means of the frequency sweep method, and b) by means of the impulse response transformation method. The reduction in measurement dynamic range is clearly noticed and is of the order of 27 dB.

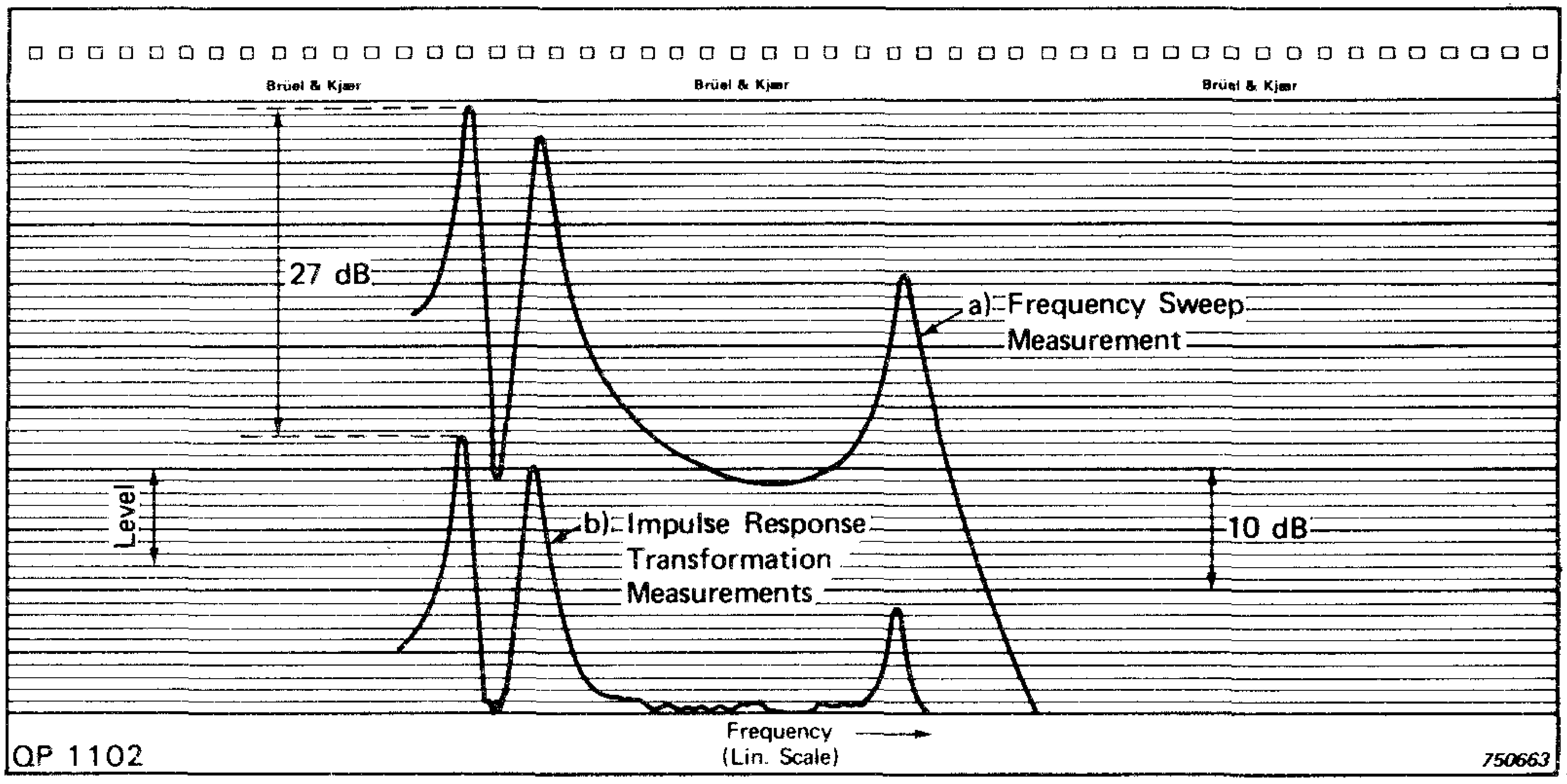

Fig.13. The frequency response function shown in Fig.12 measured a) by means of the frequency sweep technique, and b) by means of the impulse response transformation method

That a reduction of this order of magnitude was to be expected can also be estimated using the technique described above. From Fig. 13 the following values are found for the parameters involved:

$$F(f_{01}) = 1; \quad \gamma_2 = \frac{F(f_{02})}{F(f_{01})} \frac{\Delta f_2}{\Delta f_1} = 0{,}75 \frac{70}{47} = 1{,}12$$

$$\gamma_3 = \frac{F(f_{03})}{F(f_{01})} \frac{\Delta f_3}{\Delta f_1} = 0{,}2 \frac{100}{47} = 0{,}425$$

$$X_{tot.peak} = X_{1peak} (1 + 1{,}12 + 0{,}425) = 2{,}545 \cdot X_{1peak}$$

whereby

$$20 \log_{10} \left(\frac{X_{tot.peak}}{X_{1peak}} \right) \approx 8\,\text{dB}$$

If the first resonance only had been present in the test device then:

$$F(f_{01}) = X_{peak} \frac{1}{\pi} \frac{\Delta f_M}{\Delta f_1} = X_{peak} \frac{1}{\pi} \frac{12{,}5}{47} = 0{,}085$$

i. e. a loss in dynamic range compared to the frequency sweep test of $20\log_{10}$ (0,085) = 23 dB would have been obtained (Δf_M = 12,5 Hz). Due to the existence of the other resonances this loss is further increased to 23 + 8 = 31 dB, a figure somewhat larger than the actually measured value of 27 dB.

Although the estimation technique developed here seems to give a somewhat high value for the actual loss in dynamic range it clearly demonstrates the existence of this loss and allows, as mentioned, a *rough* estimate to be made of the order of magnitude of the loss.

If the resonances in the device being investigated are closely coupled, showing a more "flat" spectral "peak" than those shown in Figs. 12 and 13 (see Fig. 14) the peak can no longer be treated as if it was due to a single degree-of-freedom system response. In such cases the spectral "peak" may be approximated by a "box", Fig. 14. The "box" may be assumed to have a "height" equal to $F(f_o)$ and a width roughly equal to the —3 dB bandwidth, B. Applying a unit pulse to such a "box" would give a time-function output of the kind:

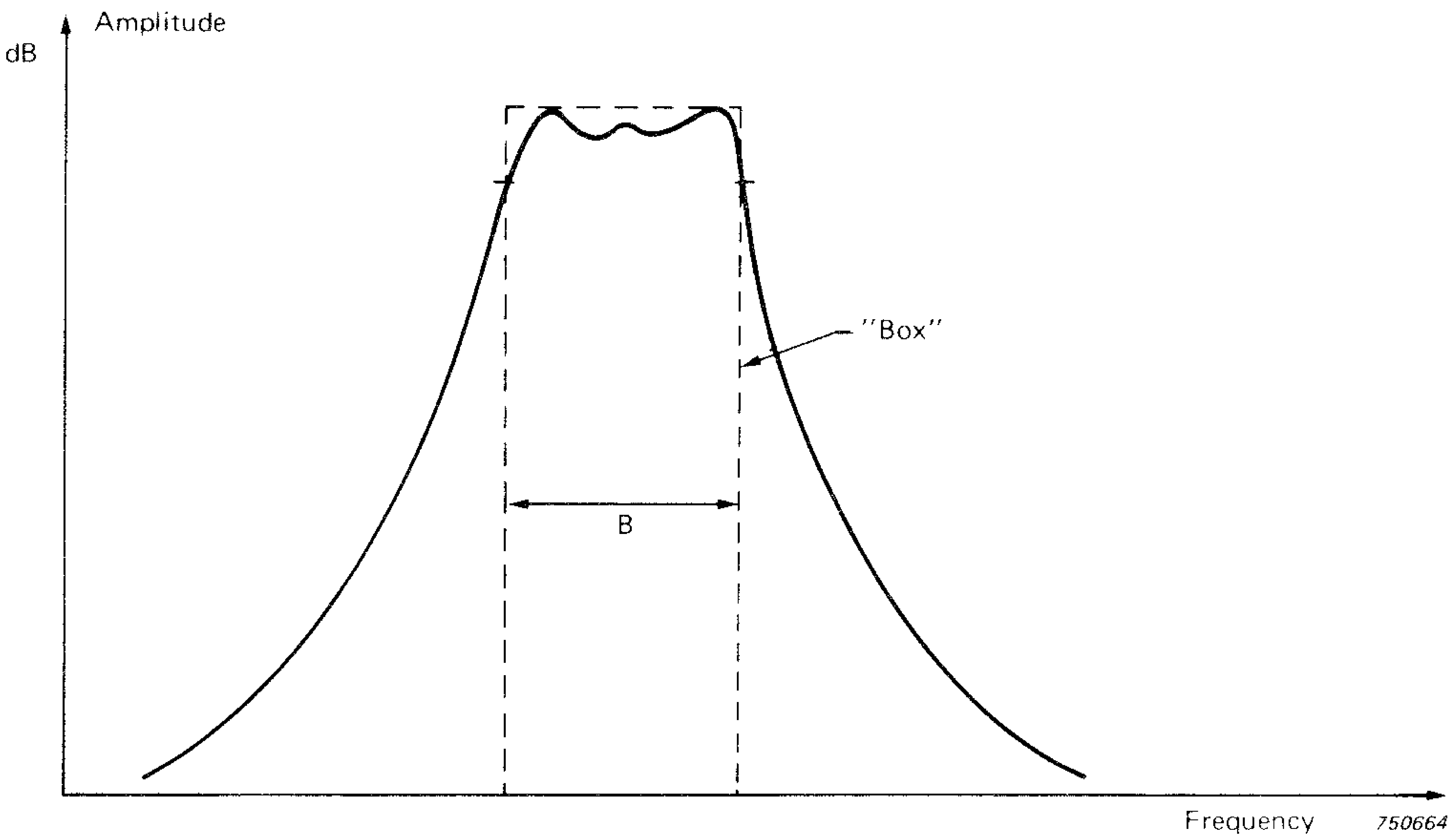

Fig. 14. Illustration of how an approximately flat spectral "peak" may be approximated by a "box"

$$X_{out}(t) = X_{peak} \frac{\sin[\pi B(t - t_L)]}{\pi B(t - t_L)} \cos(\omega_0 t)$$

as indicated in Fig. 15.

As shown in Appendix B the relationship between $F(f_o)$ and X_{peak} is, in this case:

$$F(f_0) = \frac{X_{peak}}{T_M B} = X_{peak} \frac{\Delta f_M}{B}$$

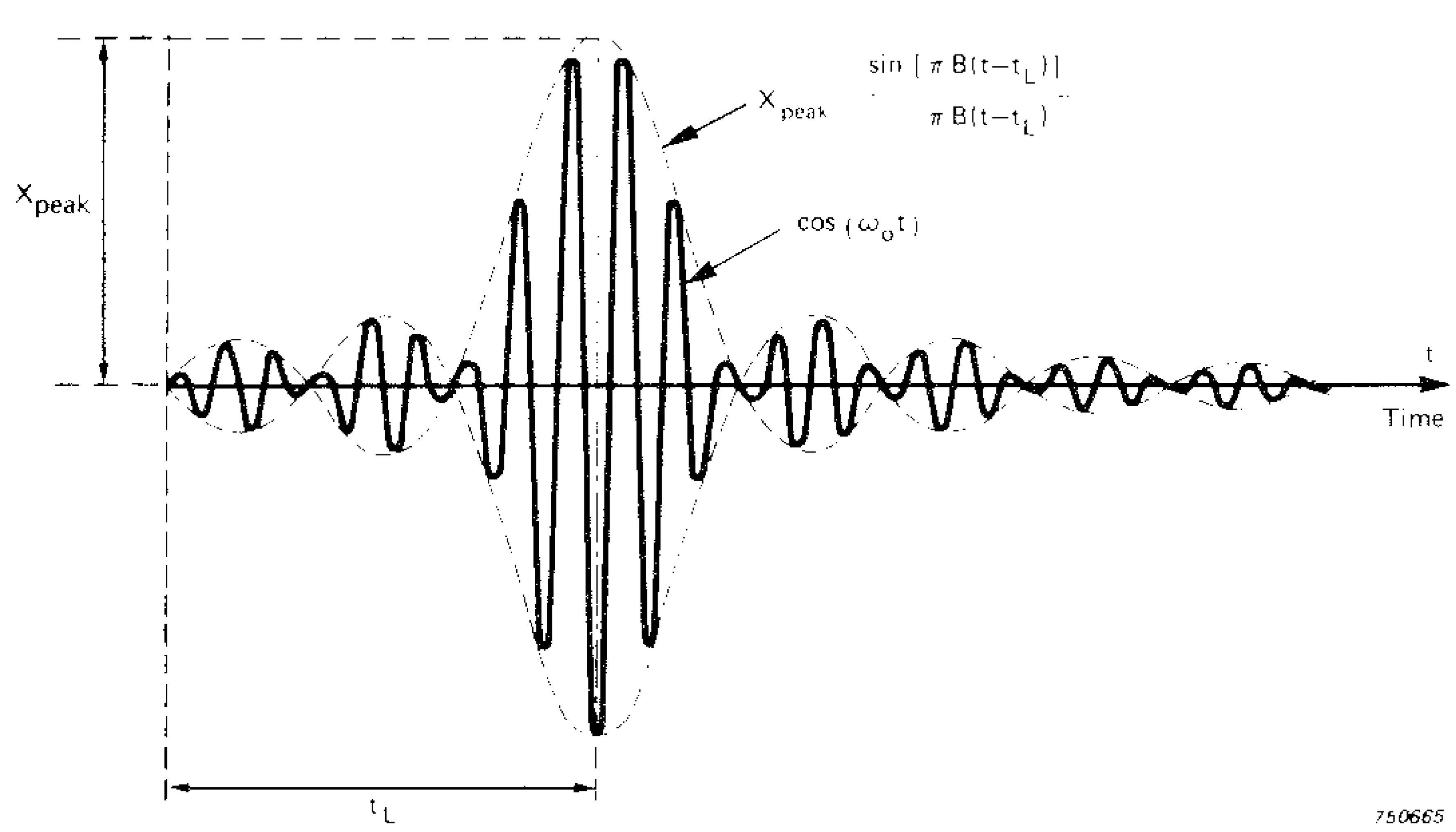

Fig. 15. Time function response of a spectral "box" to a short duration (δ-) impulse

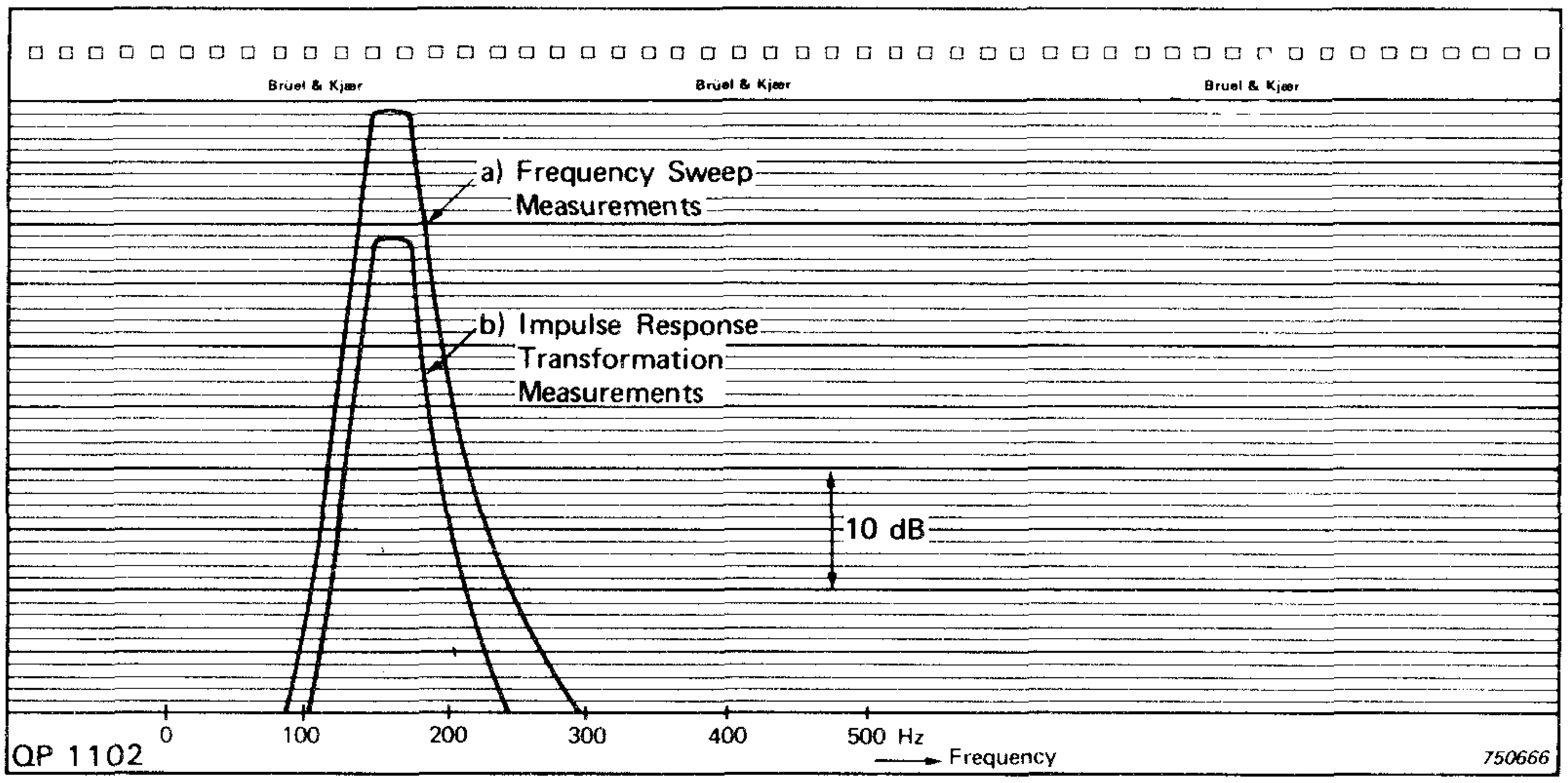

Fig.16. The frequency response function of a 1/3 octave filter measured
a) by means of the frequency sweep technique and
b) by means of the impulse response transformation method

As an example of this type of response (roughly) consider a test-device consisting of a 1/3-octave filter centered at 160 Hz. The frequency response function of such a filter is shown in Fig.16 measured, a) by means of the frequency sweep method, and b) by means of the impulse response transformation method. Note that the reduction in dynamic range is in this case only of the order of 10 dB when $3\Delta f_M = B$, which fits, very closely, with estimation results obtained by means of the above expression.

This case ("flat peak") may also be used to estimate the loss in dynamic range when wide-band test devices are considered. If, for instance, the bandwidth, B, of the device being tested was of the order of say, $400 \times \Delta f_M$ then a *loss in dynamic range of the order of 52 dB would occur.* Actual measurements confirm that the loss in this case is of the order of 45 — 50 dB, which is in fair agreement with the theoretical prediction.

So far dynamic range considerations.

The second, technically important, question to be discussed in conjunction with the use of impulse response transformation techniques is that of the measurement time required. This is, actually, fairly simple to estimate:

Assuming that a frequency resolution of 3 $\Delta f_M = \Delta f$ is sufficient, and that the response time of the (narrow band) real time analyzer is of the order of $T_M = 1/\Delta f_M$ then the minimum measurement time necessary for analysis is:

$$T_A = \frac{3}{\Delta f_{min}}$$

where Δf_{min} is the minimum resonant bandwidth of the test-device.

The Wide-Band Random Noise Method

The third method, sometimes used to obtain frequency response functions, consists of feeding the test-device with a wide-band random noise signal, and frequency analyzing the output, Fig. 17. The frequency analysis may be performed either by means of a real-time analyzer (or computer) as described in the previous section of this paper, or by means of a sweeping, narrow band frequency analyzer.*

The principles involved are actually those of "normal" frequency analysis of random signals and will not be dealt with here in any detail. It may, however, for the sake of comparison be of interest to briefly outline some basic considerations.

As the complete signal is present at the input to the analyzer no matter whether swept frequency analysis, or real time technique is used, the "loss" in obtainable measurement dynamic range is the same in both cases.

Upon the assumption that only very small errors occur if input signal peak values higher than, say $3\sigma_i$ (σ_i = RMS — value of the input signal

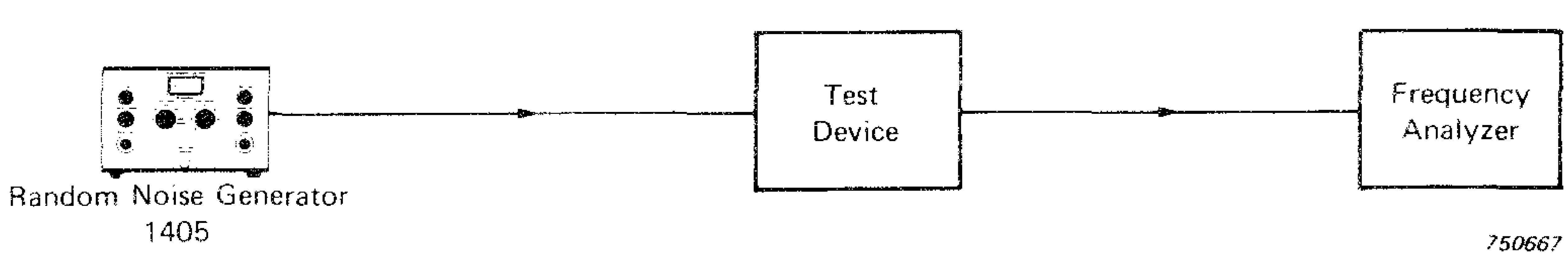

Fig. 17. Example of a measuring arrangement used in the wide-band random noise measurement method

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*) Actually also the analysis of impulse response measurements may be made in the form of sweeping narrow band frequency analysis, provided that the time-function response has been stored in an accessible storage device (magnetic tape recorder, digital event recorder, etc.)

to the analyzer) are distorted or "clipped" a general dynamic loss formula of the type:

$$F(f_1)_{max} = \frac{1}{3}\sqrt{\frac{\Delta f_M}{\Delta f_1}} \frac{1}{\sqrt{1 + \sum_{n=2}^{n} \beta_n}} X_{peak}$$

can be derived (see Appendix C). Here $F(f_1)_{max}$ is the maximum (RMS-value) measured with the analyzer (noise-) bandwidth, Δf_M, centered at the frequency f_1, Fig. 18, and X_{peak} is the maximum (peak) value of the input time function to the analyzer.

Δf_1 is, as can be seen from the figure, the equivalent energy (noise) bandwidth of the frequency response "peak" centered at f_1.

β_n is given by: $\beta_n = \frac{W_n \Delta f_n}{W_1 \Delta f_1}$

where W_n and W_1 are mean square (power) spectral densities and the Δf_n s are equivalent noise bandwidths of the corresponding frequency response function "peaks" (Fig. 18).

If the above formula is used to estimate the "loss" in dynamic range taking place when measurements with wideband random noise are made on a device with a frequency response function as shown in Fig. 12, the result is of order of 17 dB ($\Delta f_M / \Delta f_1 = 1/3$). This corresponds, very closely, to actually measured results.

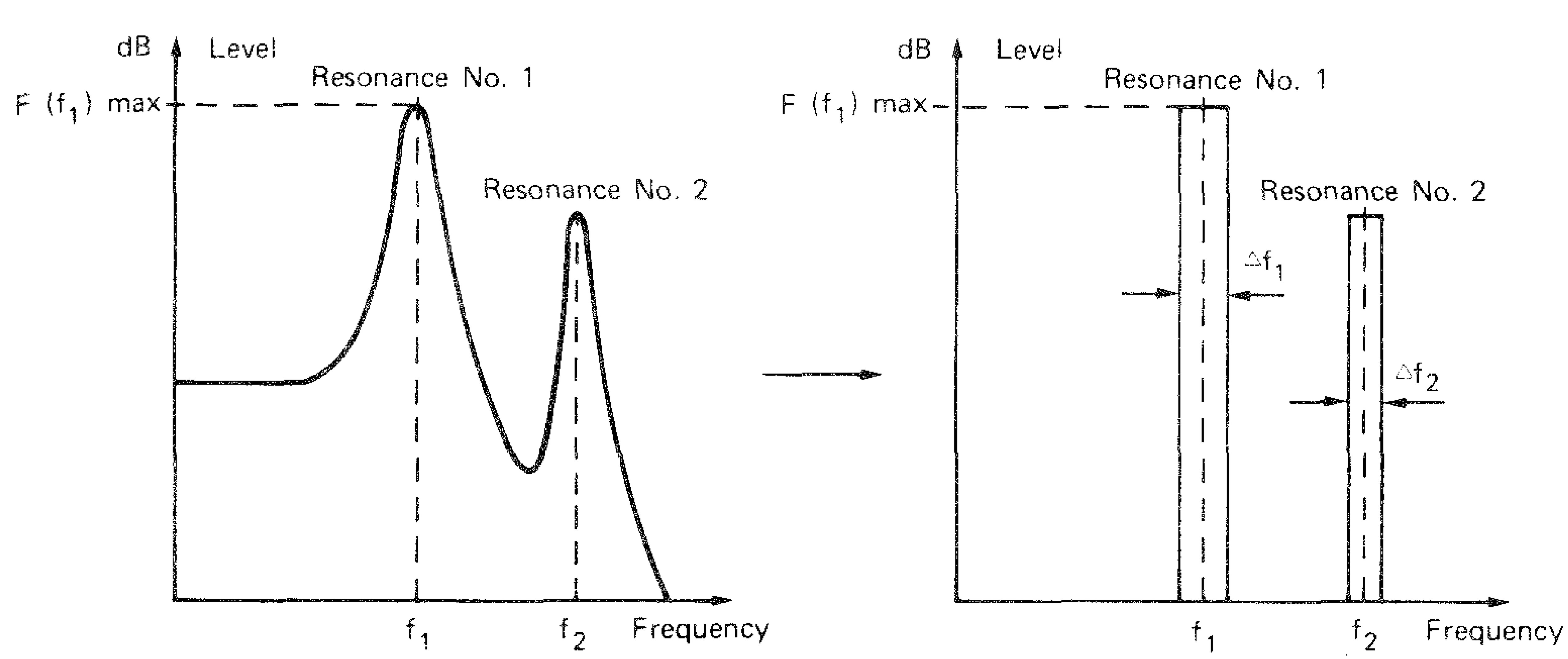

Fig. 18. Sketch of the frequency response function for a two degrees-of-freedom system and its transformation into frequency- "boxes" with equivalent (noise) energy bandwidths

Furthermore, if the input signal to the analyzer has a flat frequency spectrum ($W_n = W_1$) over a bandwidth $B = n \times \Delta f_1$ then the loss in measurement dynamic range can be seen to equal:

$$\text{"Loss" in Dynamic Range} = \frac{1}{3}\sqrt{\frac{\Delta f_M}{B}}$$

Again, actually measured results compare well with theoretical estimates

(Example: $\Delta f_M / B = 1/400$, "loss" in dynamic range: 32 dB)

With regard to measurement time considerations the required measurement time depends upon the statistical accuracy desired from the result, and will, of course, differ widely whether real time or frequency sweep analysis technique is used for the measurements.

The statistical error in a random noise measurement is given by

$$\epsilon = \frac{1}{2\sqrt{T_A \, \Delta f_M}}$$

where T_A is the averaging time employed, and Δf_M is, as before, the (narrow) measurement bandwidth. It is readily seen that to obtain a statistical accuracy of, say 5% then $T_A \times \Delta f_M$ must be of the order of 100. If the measurement bandwidth is of the order of 10 Hz then, to obtain the above mentioned accuracy an averaging time of the order of 10 seconds is required. If real time analysis is used in the measurements this will also be the totally required measurement time, while if swept analysis technique is used the total measurement time may be several hundred times the 10 seconds!

Some Other methods of Frequency Response Measurements
General Conclusions

Under certain circumstances the use of one or the other of the "basic" methods of determining frequency response functions, discussed in the preceding text, may be inconvenient or impractical.

Other, more particular methods, have therefore, from time to time appeared in the literature.

One of these methods deserves special mention. This is the method of transient testing using what is termed "a rapid frequency sweep". Here an attempt has been made to combine the good dynamic range properties of a frequency sweep measurement with the measurement time reduction feature of the impulse response transformation technique.

As the name implies the method consists in applying to the test system a frequency sweep which is made so fast that the response of the system does not reach its steady state condition, as required by a normal frequency sweep test.

However, the method requires a similar amount of sophisticated (and expensive) measurement equipment as does the impulse response transformation method.

Another method, which has been suggested, is the use of periodically repeated impulses. As this kind of excitation produces line spectra in the frequency domain the frequency resolution in the measurement results can never become better than that corresponding to the repetition frequency of the impulses. The method will therefore, in most cases, become meaningless, or reduce itself to the impulse response transformation method.

To summarize some important properties of the most practically useful, methods of frequency response measurements dealt with in this paper Table 3 has been prepared. The table is based on systems with *equal frequency resolution* properties and may be briefly concluded as follows:

1. The *frequency sweep method* has the advantages of being relatively inexpensive, showing excellent measurement dynamic range properties, but suffer somewhat from the amount of measurement time required.

2. The *impulse response transformation method* has the advantage of minimizing the required measurement time, but suffer somewhat in measurement dynamic range properties, — and requires rather sophisticated (and expensive) measurement equipment.

3. The *wide band random noise method* may be generally applicable (even on systems in normal operation) but is time consuming, shows limited dynamic range properties and requires sophisticated (and expensive) measurement equipment.

Measurement Method	Test Conditions	Measurement Dynamic Range	Measurement Time Required	Amount of Instrumenation Required
Point by Point Method		Excellent	Very Long	Small (Very low cost method)
"Normal" Frequency Sweep Method		Excellent	Medium	Small (low cost)
Impulse Response Transformation Method	Series Analysis	Seriously Reduced (Depends on Test Device)	Relatively Long	Reasonable
	Parallel Anal. (Real Time)	—— " ——	Very Short	Considerable (Expensive Method)
Wide Band Random Noise Method	Series Anal.	Reduced (Depends on Test Device)	Long	Reasonable
	Parallel Anal. (Real Time)	—— " ——	Medium to Long	Considerable (Expensive)
Rapid Freq. Sweep Method	Series Anal.	Good	Relatively Long	Considerable
	Parallel Anal. (Real Time)	—— " ——	Short	Considerable

Table 3.

4. The *rapid frequency sweep method* has the advantage of combining reasonably short measurement time with reasonably good dynamic range properties. It requires, however, relatively sophisticated (and expensive) measurement equipment.

The preceeding text has been concentrated on the measurement of frequency response function modulii. Also, the measurement of phase is possible, although it requires some additional measurement equipment. It does not, however, impose any serious technical limitations to what has already been discussed, except with respect to phase response of the measurement system. For reasons given in the introduction to the paper, however, it has not been found relevant to go into deeper discussions here on the subject.

Before closing this discussion on frequency response measurements it should be mentioned that an "optimum" frequency sweep measurement system would seem to be one in which the sweep function is controlled by the actual steepness of response level changes. Such a system would combine excellent measurement dynamic properties with *reasonably* short measurement times.

Acknowledgement
The author is greatly indebted to Mr. C. G. Wahrmann for many valuable discussions during the development of this paper.

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APPENDIX A

Effects of Detector Averaging Time Upon the Maximum Allowable Frequency Sweep Speed

The basic relationships between the detector averaging time and the "errors" occurring in measurements were discussed in some details in the B & K Technical Reviews Nos. 2 and 3—1975, and only a brief outline of the major results, as related to frequency sweep measurements are outlined below. It was found (T.R.3) that the errors, ξ_{x1} in a swept resonance response maximum (and minimum) were given by expressions of the form:

$$\xi_x = F_x\left(\frac{S \cdot T_{av}}{\Delta f}\right)$$

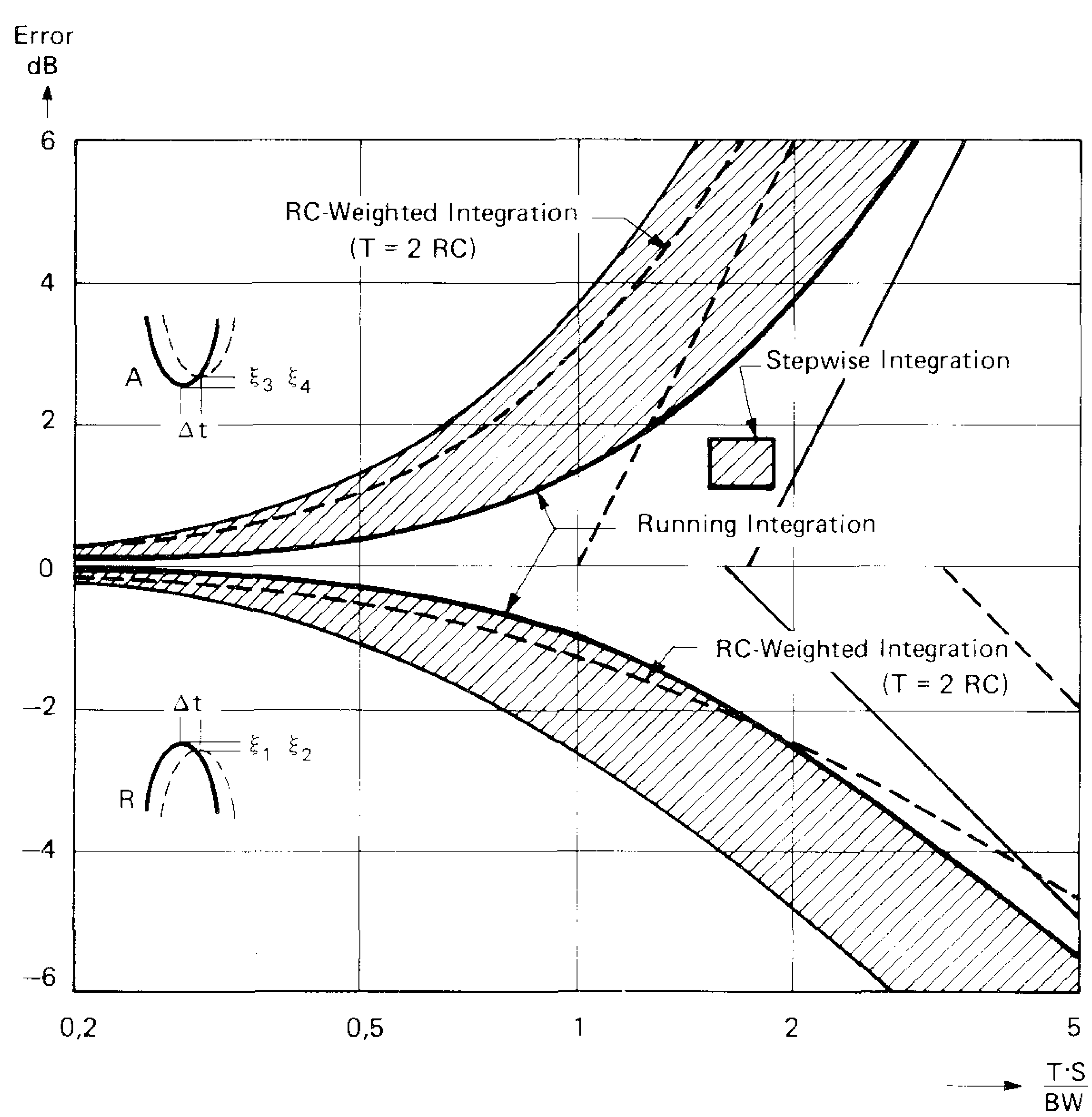


Fig.A.1. Error curves for swept resonance responses (amplitudes) when various kinds of averaging techniques are used

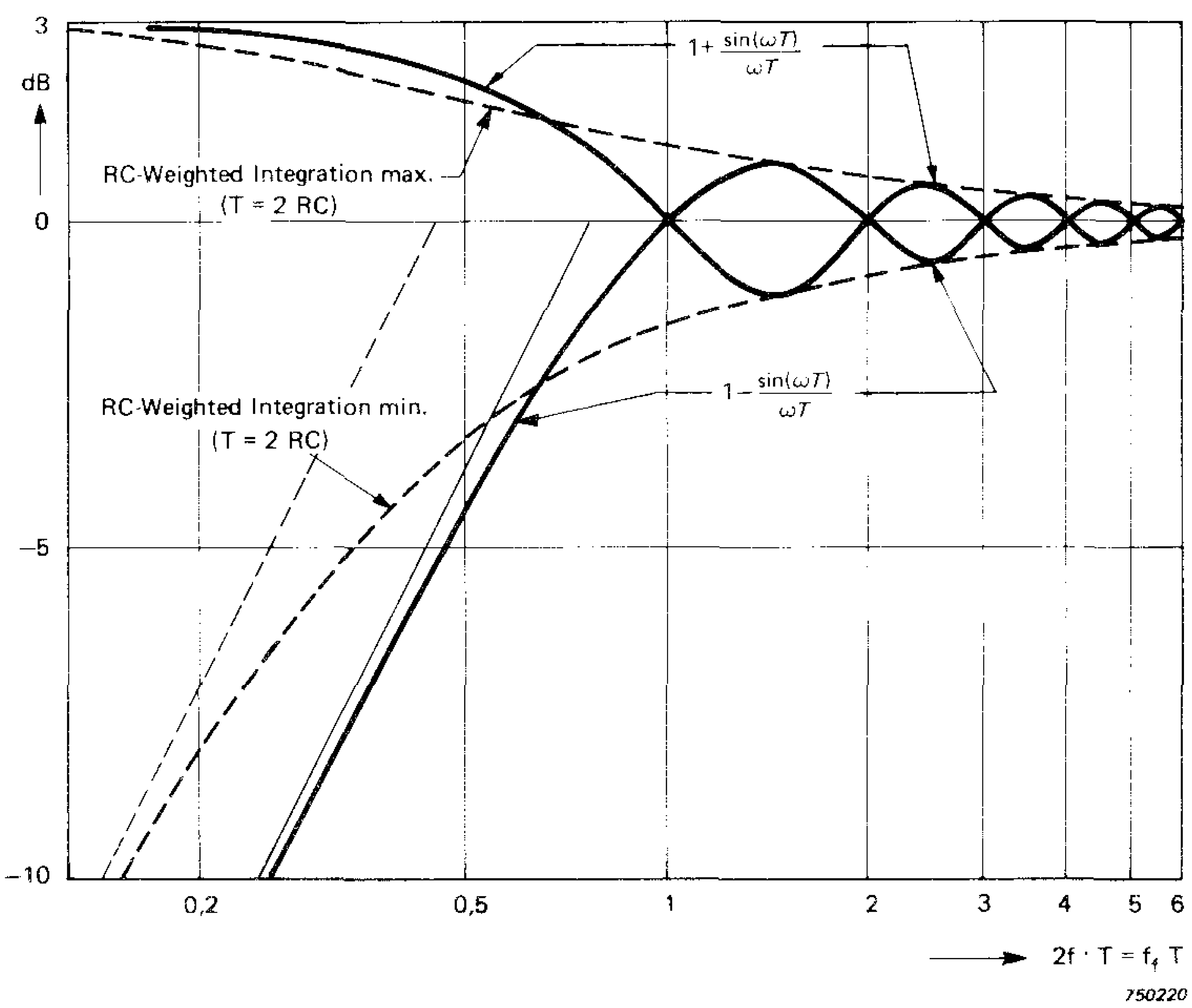


Fig.A.2. Maximum "ripple" of sine wave signals after detection and averaging

where the functions, F_x, depended upon what kind of physical averaging system was used and whether the response was a maximum or a mimimum. The functions were plotted graphically and are reproduced in Fig. A.1.

From the figure it can be deduced that the error, ξ_{x1} is of the order of 0,5 dB or less when

$$\frac{S \cdot T_{av}}{\Delta f} \leqq 0.3$$

where S and Δf represent the sweep speed and the resonance half-power bandwidth, respectively, and T_{av} is the effective averaging time in seconds†).

Furthermore, it was found (T.R. 2) that to reduce "ripple" in the time averaged signal level certain relationships existed between the signal frequency, f, the effective averaging time, T_{av}, and the "ripple" ε_x/E see Fig. A.2.

Thus, from Fig. A.2 a "ripple-condition" of the kind:

$$f \cdot T_{av} \geqq 3$$

can be formulated.

† If RC-averaging is used then $T = 2RC$.

When time-averaging is used in the detector circuit the two above expressions should be fulfilled in addition to the "general" sweep formula:

$$S \leqq (\Delta f)^2$$

Setting $S = (\Delta f)^2$ one obtains the conditions:

$$\Delta f \cdot T_{av} \leqq 0{,}3 \qquad \text{and} \qquad f \cdot T_{av} \geqq 3$$

and with $\Delta f = f/Q$ then

$$\Delta f \cdot T_{av} \leqq 0{,}3 \qquad \text{gives} \qquad f \cdot T_{av} \leqq 0{,}3Q$$

It is readily seen that the two "additional" conditions can only be fulfilled when

$$Q \geqq 10$$

If $Q \leqq 10$ then the "governing" condition is:

$$f \cdot T_{av} = 3$$

and the corresponding maximum sweep speed is given by:

$$S \leqq \frac{0{,}3 \cdot \Delta f}{T_{av}}$$

On the basis of the above derived expressions the time required for a "full" frequency sweep can be determined, also for cases of low-Q resonances in the test device.

A case, which has not been treated so far, and which is quite important in practice, is *the case where the averaging time during a frequency sweep remains constant and the frequency sweep function is logarithmic.* Actually, under these conditions Q must remain constant (not increase with the square root of frequency).

The required sweep time, however, does not change.

A table indicating the various relationships between the response characteristics of the device under test, the averaging time and the frequency sweep characteristics is given overleaf.

Frequency Sweep Function	Test Device Res. Characteristics		Averaging Time	Frequency Sweep Time
Hyperbolic	$Q \leqslant 10$	$Q = \text{Const.}$	Variable: $T_{av} = \frac{3}{f}$	$T = \frac{10\,Q}{f_L}\left(1 - \frac{f_L}{f_H}\right)$
	$Q > 10$		Variable: $T_{av} = \frac{0,3}{f}\,Q$	$T = Q^2\left(\frac{1}{f_L} - \frac{1}{f_H}\right) = \frac{Q^2}{f_L}\left(1 - \frac{f_L}{f_H}\right)$
Logarithmic	$Q \leqslant 10$	$Q = C_x\sqrt{f}$	Variable: $T_{av} = \frac{3}{\sqrt{f_L\,f}}$	$T = \frac{10\,C_x}{f_L}\ln\left(\frac{f_H}{f_L}\right)$
		$Q = \text{Const.}$	Constant: $T_{av} = \frac{3}{f_L}$	$T = \frac{10\,Q_L}{f_L}\ln\left(\frac{f_H}{f_L}\right)$
	$Q > 10$	$Q = C_x\sqrt{f}$	Variable: $T_{av} = \frac{0,3}{f}\,Q$	$T = C_x^{\,2}\ln\left(\frac{f_H}{f_L}\right) = \frac{Q_L^{\,2}}{f_L}\ln\left(\frac{f_H}{f_L}\right)$
		$Q = \text{Const.}$	Constant: $T_{av} = \frac{0,3}{f_L}\,Q_L$	
Linear	$Q \leqslant 10$	$Q = C_y\,f$	Constant: $T_{av} = \frac{3}{f_L}$	$T = 10\,C_y\,(f_H - f_L) = \frac{10\,Q_L}{f_L}\left(\frac{f_H}{f_L} - 1\right)$
	$Q > 10$		Constant: $T_{av} = \frac{0,3}{f_L}\,Q_L$	$T = C_y^{\,2}\,(f_H - f_L) = \frac{Q_L^{\,2}}{f_L}\left(\frac{f_H}{f_L} - 1\right)$

APPENDIX B

Spectral Analysis of Transients

The frequency spectrum of a transient time function can be mathematically obtained by applying the Fourier integral theorem to the problem.

On the other hand, if the transient is captured in the memory of an instrument (computer) and continuously recirculated the transient becomes a periodic signal with a period equal to the recirculation time, and a wave-shape equal to the transient time function. Spectral analysis of this kind of periodic signals can be made by means of Fourier series theorem, with the coefficients:

$$A_n = \frac{2}{T_M}\int_0^{T_M} X(t)\cos\left(\frac{2\pi n}{T_M}\cdot t\right) dt$$

$$B_n = \frac{2}{T_M}\int_0^{T_M} X(t)\sin\left(\frac{2\pi n}{T_M}\cdot t\right) dt$$

where T_M is the recirculation time, $X(t)$ is the transient time function, and n is an integer.

Consider next the two transient time functions:

$$X_1(t) = X_{1\ \text{peak}} \cdot \exp\left(-\frac{\omega_0}{2Q}t\right) \cdot \cos(\omega_0 t)$$

and

$$X_2(t) = X_{2\ \text{peak}} \cdot \frac{\sin(\pi B(t-t_L))}{\pi B(t-t_L)} \cos(\omega_0 t)$$

The two functions and their respective Fourier transforms are indicated in Fig. B.1. If these two transients were captured and analyzed separately as periodic signals it is clear that the maximum value of the Fourier series coefficients would occur when $2\pi n/T_M = \omega_0$ (see Fig. B.1.). Furthermore, it is a "criterion" for the periodic analysis that the transient has died out (practically) within the recirculation period, T_M. Therefore no serious error is introduced by replacing the integration limits 0 to T_M in the expressions given above for A_n and B_n by the limits 0 to ∞.

Considering first the function $X_1(t)$:

$$A_{n1\ \max} = \frac{2}{T_M}\int_0^{\infty} X_{1\ \text{peak}} \cdot \exp\left(-\frac{\omega_0}{2Q}t\right) \cdot \cos^2(\omega_0 t)\, dt$$

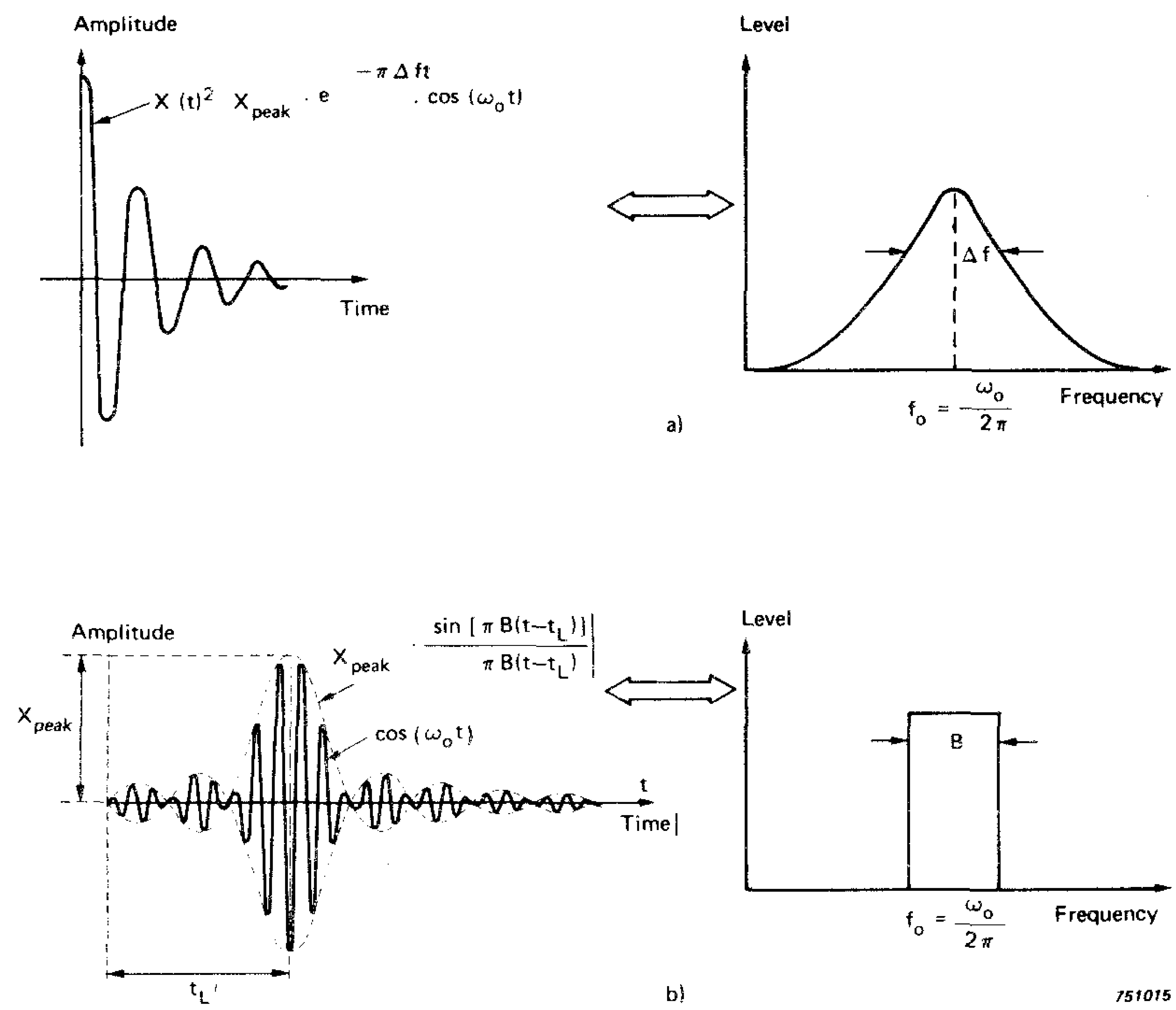


Fig.B.1. Response time functions and their respective Fourier transforms
a) Exponentially decaying resonant transient
b) "Idealized" response transient

From integral tables it is found that:

$$I_1 = \int_0^\infty \exp(-at) \cdot \cos^2(mt)\, dt = \frac{a^2 + 2m^2}{a(a^2 + 4m^2)}$$

and when $Q \gg 1$ then $a = \dfrac{\Delta\omega}{2} \ll \omega_0$ and $I_1 \approx \dfrac{1}{2a} = \dfrac{1}{\Delta\omega}$ whereby:

$$A_{n1\ max} \approx \frac{2X_{1\ peak}}{T_M \cdot \Delta\omega}$$

Similarly, with the same assumptions as above:

$$B_{n1\ max} = \frac{2}{T_M} \int_0^\infty X_{1\ peak} \cdot \exp\left(-\frac{\omega_0}{2Q}t\right) \cdot \cos(\omega_0 t)\sin(\omega_0 t)\, dt$$

$$\approx \frac{X_{1\ peak}}{2T_M\omega_0} \ll A_{n1\ max}$$

so that $\quad A_{n1\ max} = F_1(fo) = 2\,\dfrac{X_{1\ peak}}{T_M \cdot \Delta\omega} = 2X_{1\ peak} \cdot \dfrac{\Delta f}{\Delta\omega}$

because, as stated in the main text, $\Delta f_M = 1/T_M$.

Considering next the function $X_2(t)$. Here a "translation" of the time-function is most convenient, setting $t_L = 0$. Since the function will then be symmetrical around the Y-axis the required integral is obtained by integrating from 0 to ∞ and multiplying the result by 2:

$$A_{n2\,\max} = 2 \cdot \frac{2}{\alpha T_M} \cdot X_{2\text{peak}} \int_0^{\infty} \frac{\sin(\alpha t)\cos^2(mt)}{t}\,dt$$

where $\alpha = \pi B$ and $m = \omega_0$.

Again, from standard tables of integrals (with a slight modification) one obtains:

$$A_{n2\,\max} = X_{2\,\text{peak}} \cdot \frac{1}{T_M B}$$

In this case $B_{n2\,\max}$ becomes 0 and:

$$A_{n2\,\max} = F_2(f_0) = \frac{X_{2\,\text{peak}}}{T_M \cdot B} = X_{2\,\text{peak}} \frac{\Delta f_M}{B}$$

APPENDIX C

On the Response of Resonant Systems to Wide-Band Random Excitation

In the frequency domain random noise signals are commonly characterized by their mean square (power) spectral density functions. If a test device is subjected to a random noise input signal, the mean square spectral density of which is constant and independent of frequency over the complete frequency range to which the device respond, the output mean square spectral density function will be a replica of the device's squared frequency response function (modulus).

From the theory of random noise it is, furthermore, known that the *RMS*-value of the noise output signal from a filter with unity gain and an equivalent noise bandwidth Δf excited by an input signal with a constant mean square spectral density W_{in} is given by the expression:

$$\sigma_{out} = \sqrt{W_{in} \cdot \Delta f}$$

σ_{out} being the output signal *RMS*-value.

In cases where the filter has a certain (voltage) gain, G, then:

$$\sigma_{out} = \sqrt{W_{in} \cdot G^2 \cdot \Delta f}$$

Setting $W_{in} \cdot G^2 = W$ (equivalent *output* mean square spectral density) then:

$$\sigma_{out} = \sqrt{W \cdot \Delta f}$$

Assuming that a test device contains several filters (resonances), with different gains and different equivalent noise bandwidths (see *f.* inst. Fig. 18 in the main text) a general expression for the output signal *RMS*-value would be:

$$\sigma_{out} = \sqrt{\sum_n W_n \Delta f_n}$$

If this signal is applied to a frequency analyzer, as suggested in Fig. 17 of the main text, and the analyzer input allows a maximum time signal peak value of $X_{peak} = 3\sigma_{out} = 3\sqrt{\sum_n W_n \Delta f_n}$

Setting:

$$3\sqrt{\sum_n W_n \Delta f_n} = 3\sqrt{W_1} \cdot \sqrt{\Delta f_1 \left(1 + \sum_{n=2}^{n} \beta_n\right)}$$

where

$$\beta_n = \frac{W_n \cdot \Delta f_n}{W_1 \cdot \Delta f_1}$$

then:

$$\sqrt{W_1} = \frac{X_{peak}}{3\sqrt{\Delta f_1} \cdot \sqrt{1 + \sum_{n=2}^{n} \beta_n}}$$

The maximum value, $F(f_1)_{max}$ indicated by the (*RMS*) readout of the analyzer at the frequency f_1 is:

$$F(f_1)_{max} = \sqrt{W_1 \cdot \Delta f_M}$$

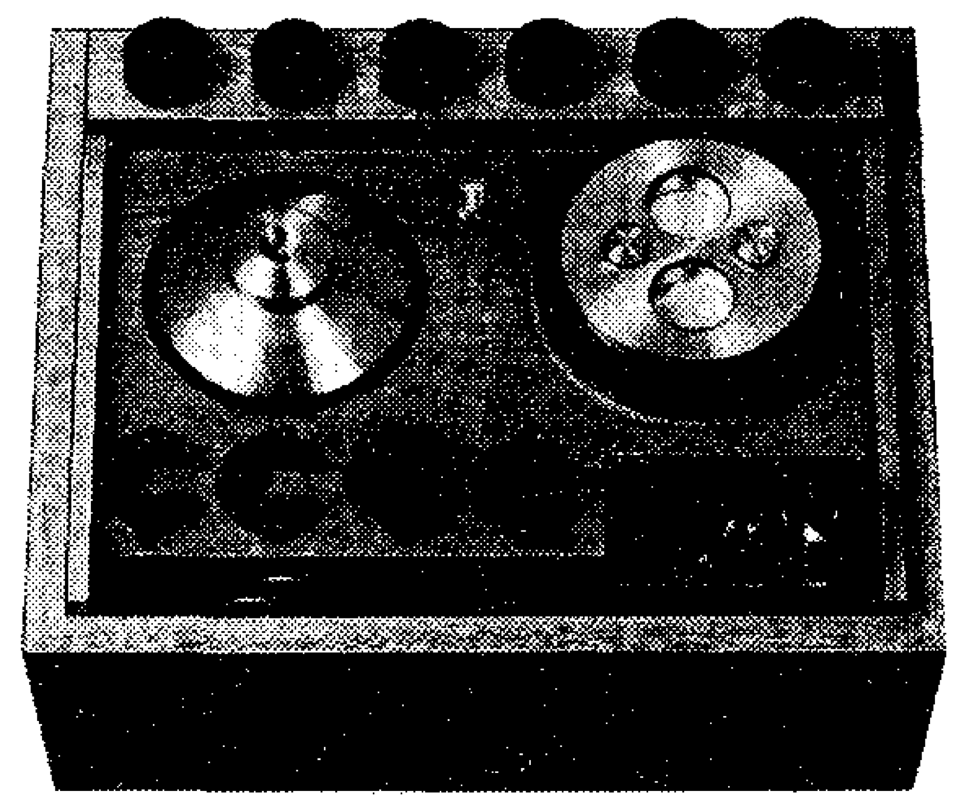
whereby the relation between the maximum time signal input peak, X_{peak}, to the analyzer, and the analyzer spectral indication, $F(f_1)_{max}$ becomes:

$$F(f_1)_{max} = \frac{1}{3}\sqrt{\frac{\Delta f_M}{\Delta f_1}} \cdot \frac{1}{\sqrt{1 + \sum_{n=2}^{n} \beta_n}} X_{peak}$$

which is the expression stated in the main text.

News from the Factory

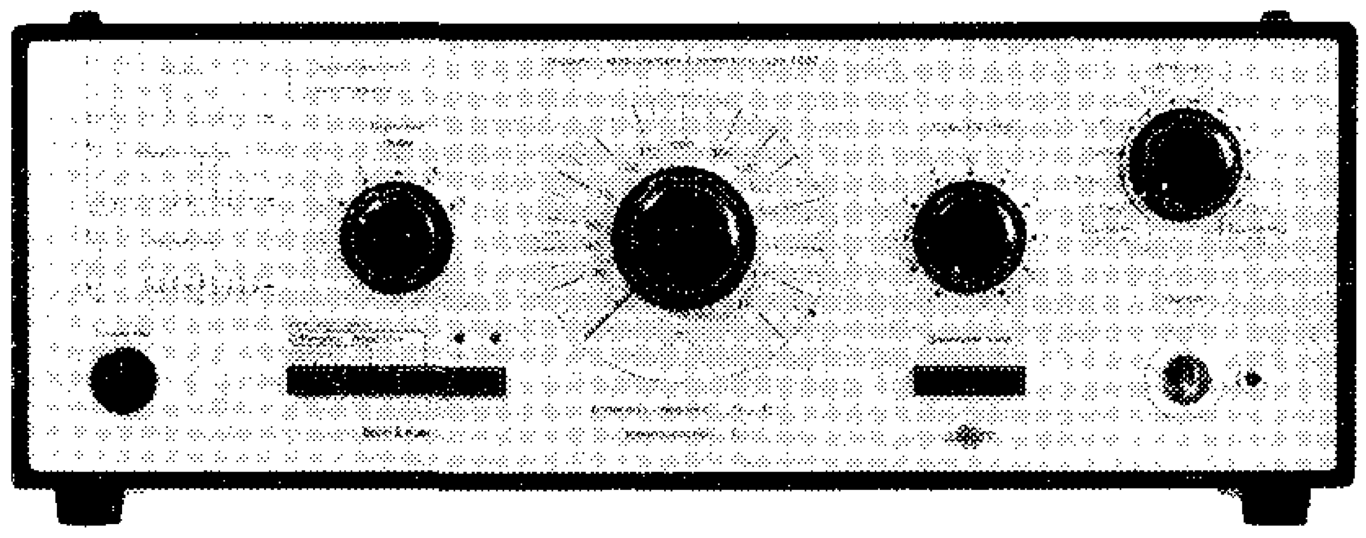
High Pressure Microphone Calibrator Type 4221



The High Pressure Microphone Calibrator Type 4221 allows both absolute and comparison calibrattion of 1", 1/2", 1/4" and 1/8" condenser microphones and other pressure transducers at dynamic pressures as high as 164 dB re. 20 μPa. Using tone bursts, however, calibration up to 172 dB can be performed. The calibrator is delivered with a high pressure coupler and a low frequency coupler with their individual calibration charts. Using the two different couplers, frequency range from 10^{-3} Hz to 1000 Hz can be covered. Depending on the frequency range the accuracy achieved is ± 1 dB or ± 1,5 dB. However, higher accuracies can be achieved either by using the individual calibration charts or utilizing a compressor loop. Typical measurements that can be made are sensitivity, frequency response, distortion and linearity.

The high sound pressure is generated in a closed coupler, having a small volume (high acoustic impedance), by a piston moved by an electromagnetic exciter. To achieve low acoustic impedance for the exciting system, the piston area is made large, the entire moving system is made with a small mass and the exciter is equipped with soft flexures. The low acoustic impedance of the exciter system compared with the high impedance of the coupler ensures that the sound pressure generated in the coupler is independent of the coupler volume, atmospheric pressure, influence of adiabatic or isothermal conditions and non-linearity in load impedance at high dynamic pressures.

Distortion Measurement Control Unit Type 1902



The Distortion Measurement Control Unit Type 1902 is an accurate instrument designed to remotely control the Heterodyne Analyzer Type 2010 for carrying out automatic swept distortion analysis. The measurements are easier to perform than the tedious method normally used for distortion measurements at several single frequencies.

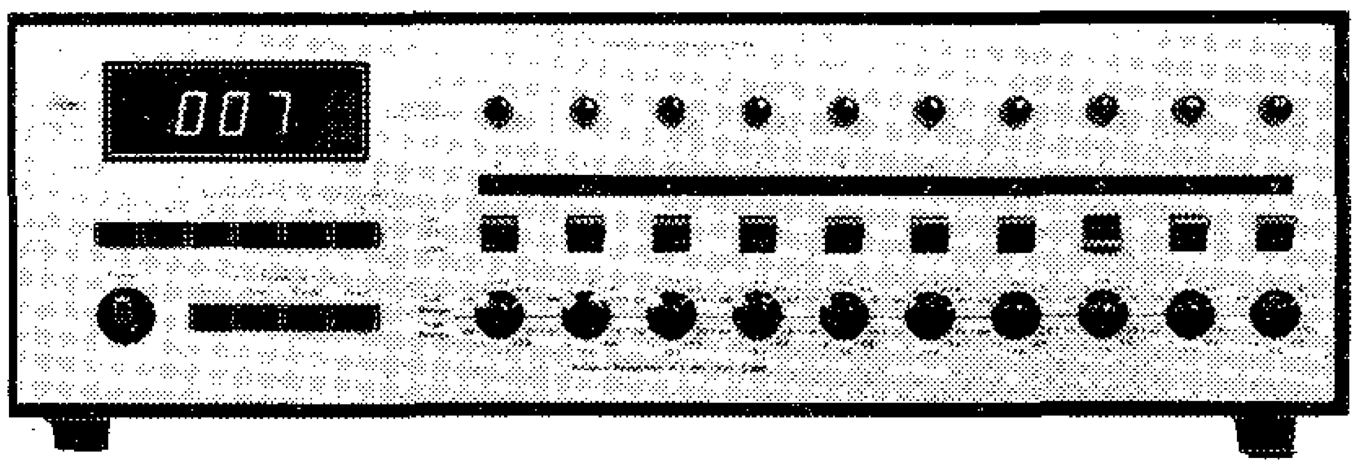
The 1902/2010 combination can measure harmonic distortion, difference frequency distortion and intermodulation distortion all to DIN 45403 and IEC 268-3 standards. Difference frequency distortion and intermodulation distortion may also be measured according to the CCIF and SMPTE methods respectively.

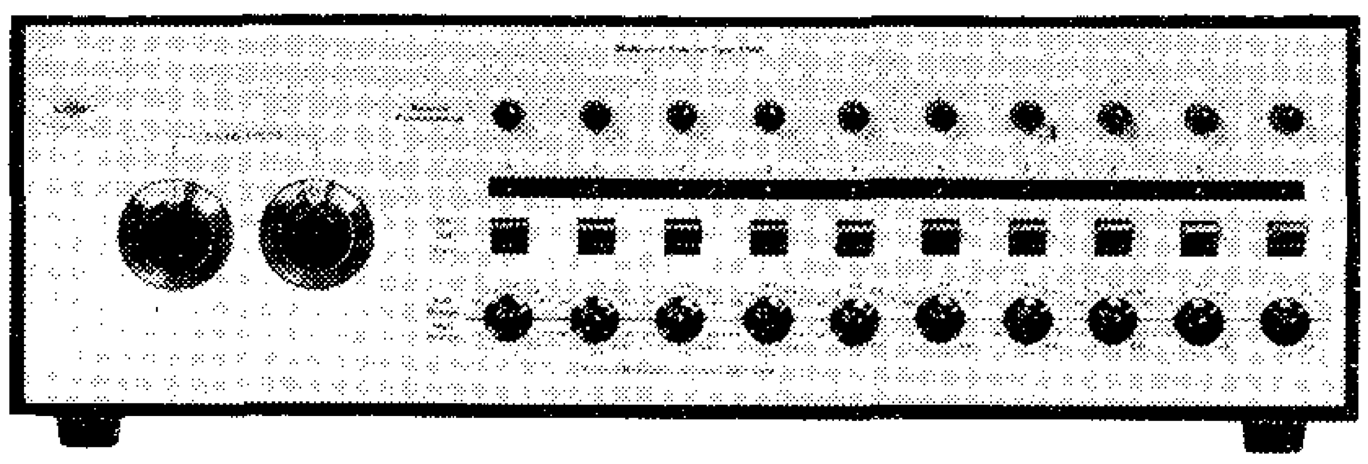
The 1902 provides tuning signals for the 2010 and generates the necessary test signals for swept measurement of distortion components up to fifth order in the frequency range 2 Hz to 200 kHz. Distortion components can be measured down to, typically, —80 dB.

The amplitude linearity is better than ± 0,2 dB while typical distortion is less than 0,01%. When the 1902/2010 combination is used together with the Level Recorder Type 2305 or 2307, automatic analysis of distortion can be performed and documented on preprinted frequency calibrated paper.

Typical fields of application are measurements on amplifiers, loudspeakers, hearing aids, tape recorders, microphones, hydrophones, etc..

Multipoint Selector and Control Type 1544 and
Multipoint Selector Type 1545





The Multipoint Selector and Control Type 1544 has provision for connecting up to 10 measuring points to the Strain Indicator Type 1526. When more than 10 points have to be measured, Multipoint Selectors Type 1545 with a provision for connection of 10 points in each, can be added to the arrangement. The Multipoint Selector and Control Type 1544 is the master control unit which can connect up to 39 Multipoint Selectors via itself to the Strain Indicator giving a total of 400 measuring point capability. The 1544 and 1545 have individual Bridge Mode and Balance controls for each measuring point and any combination of full, half or quarter (with adaptor) bridge can be connected.

The measuring points can either be stepped manually, remotely or automatically scanned with step intervals selectable between 0,1 s and 10 s. The Multipoint Selector and Control features a digital display to indicate which measuring point is connected for measurement, while the Multipoint Selector has a switchable decade selector. Individual measuring points can be switched out of the scan. A single measuring point can be called for measurement or balancing, and if several points are called a restricted automatic sweep of the chosen points can be made.

Finally, the 1544 has a built-in interface to the B & K Data Bus and a data output of strain level and point identity.

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(Continued from cover page 2)

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